Énoncés des exercices « 3s - Dérivées II: variations et asymptotes »

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3s - Dérivées II : variations et asymptotes - Corrigés

Corrigé de l'exercice 1

$$D_f : \; x^2 + 4x + 3 \neq 0$$
$$D_f = \mathbb{R} \backslash \{-3; -1\}$$

$$f(x) = \frac{(x+1)(x-2)}{(x+3)(x+1)} = \frac{x-2}{x+3}$$
$$\lim_{x \to -1} \frac{x-2}{x+3} = \frac{-1-2}{-1+3} = -\frac{3}{2}$$

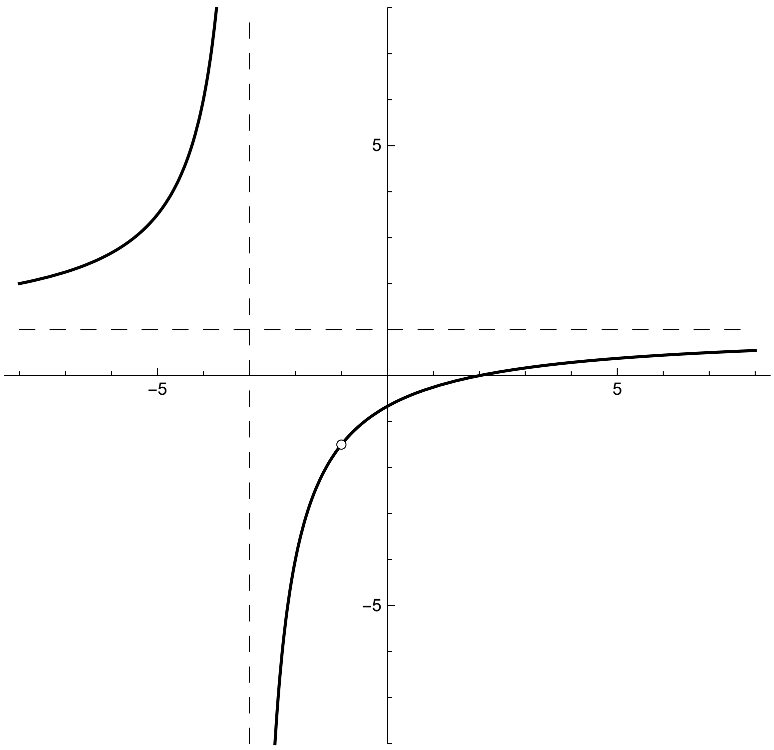
Il y a un trou en $\left(-1; -\frac{3}{2}\right)$.

$$\lim_{x \uparrow -3} \frac{x-2}{x+3} = \frac{-5}{0^-} = +\infty$$
$$\lim_{x \downarrow -3} \frac{x-2}{x+3} = \frac{-5}{0^+} = -\infty$$

Asymptote verticale double $x = -3$.

$$\lim_{x \to \pm\infty} \frac{x-2}{x+3} = 1$$

Asymptote horizontale double $y = 1$.



Corrigé de l'exercice 2

$$D_f = \mathbb{R}\backslash\{-2\}$$

$$\lim_{x\uparrow -2} f(x) = +\infty$$

$$\lim_{x\downarrow -2} f(x) = -\infty$$

Asymptote verticale double $x = -2$.

$$a = \lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{3 - x^2}{2x^2 + 4x} = \lim_{x \to \pm\infty} \frac{-x^2}{2x^2} = -\frac{1}{2}$$

$$\begin{aligned} b &= \lim_{x \to \pm\infty} (f(x) - ax) = \lim_{x \to \pm\infty} \left(\frac{3 - x^2}{2x + 4} + \frac{1}{2}x\right) \\ &= \lim_{x \to \pm\infty} \frac{3 - x^2 + x(x+2)}{2x + 4} = \lim_{x \to \pm\infty} \frac{2x + 3}{2x + 4} = 1 \end{aligned}$$

Asymptote oblique double $y = -\frac{1}{2}x + 1$

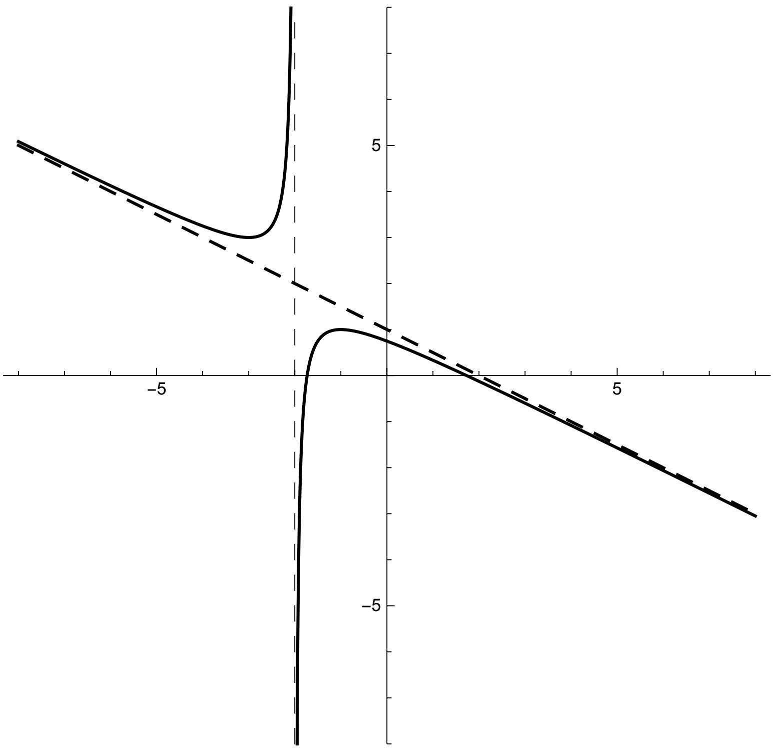
$$f'(x) = \frac{1}{2}\frac{-2x \cdot (x+2) - (3 - x^2) \cdot 1}{(x+2)^2} = \frac{1}{2}\frac{-x^2 - 4x - 3}{(x+2)^2}$$

$$Z_{f'} = \{-3, -1\}$$

x	$-\infty$		-3			-2			-1		∞
$\text{sign}(-x^2 - 4x - 3)$		$-$	0	$+$		$+$		$+$	0	$-$	
$\text{sign}(f'(x))$		$-$	0	$+$		$\|$		$+$	0	$-$	
$\text{Var}(f(x))$	∞				∞	$\|$			1		
		$\searrow$		$\nearrow$		$\|$		$\nearrow$		$\searrow$	
			3			$\|$	$-\infty$				$-\infty$

$$\text{Min : } f(-3) = 3$$

$$\text{Max : } f(-1) = 1$$



Corrigé de l'exercice 3

$$D_f = \mathbb{R}\backslash\{-2\}$$

$$\begin{aligned} f'(x) &= \frac{2\left(x^2-9\right)2x(x+2)-\left(x^2-9\right)^2 1}{(x+2)^2} \\ &= \frac{\left(x^2-9\right)\left(4x^2+8x-\left(x^2-9\right)\right)}{(x+2)^2} \\ &= \frac{\left(x^2-9\right)\left(3x^2+8x+9\right)}{(x+2)^2} \end{aligned}$$

$$Z_{f'} = \{-3; 3\}$$

x	$-\infty$	-3		-2		3	∞
$\text{sign}(x^2-9)$	$+$	0	$-$	$-$	$-$	0	$+$
$\text{sign}(3x^2+8x+9)$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$\text{sign}\left((x+2)^2\right)$	$+$	$+$	$+$	0	$+$	$+$	$+$
$\text{sign}(f'(x))$	$+$	0	$-$	$\Vert$	$-$	0	$+$
		0		$\Vert$	∞		∞
$\text{Var}(f(x))$	$\nearrow$		$\searrow$	$\Vert$	$\searrow$		$\nearrow$
	$-\infty$		$-\infty$	$\Vert$		0	

$$f(-3) = 0; \quad \text{maximum en } (-3; 0)$$
$$f(3) = 0; \quad \text{minimum en } (3; 0)$$

Corrigé de l'exercice 4

$$\begin{aligned} D_f: \quad & x^3 + x^2 - 6x \neq 0 \\ & x(x+3)(x-2) \neq 0 \\ & D_f = \mathbb{R}\backslash\{-3; 0; 2\} \end{aligned}$$

$$f(x) = \frac{x^2-4}{x^3+x^2-6x} = \frac{(x-2)(x+2)}{x(x+3)(x-2)} = \frac{x+2}{x(x+3)}$$

$$\lim_{x \to 2} \frac{x+2}{x(x+3)} = \frac{2+2}{2(2+3)} = \frac{2}{5}$$

Il y a un trou en $\left(2; \frac{2}{5}\right)$.

$$\lim_{x \uparrow 0} \frac{x+2}{x(x+3)} = \frac{2}{(0^-)\,3} = -\infty$$

$$\lim_{x \downarrow 0} \frac{x+2}{x(x+3)} = \frac{2}{(0^+)\,3} = +\infty$$

Asymptote verticale double $x = 0$

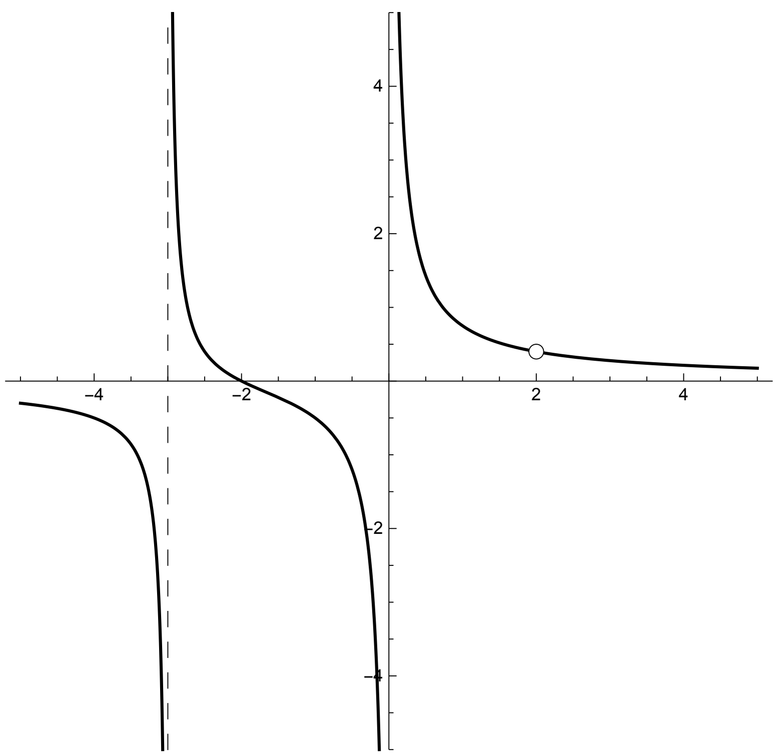
$$\lim_{x \uparrow -3} \frac{x+2}{x(x+3)} = \frac{-1}{(-3)0^-} = -\infty$$

$$\lim_{x \downarrow -3} \frac{x+2}{x(x+3)} = \frac{-1}{(-3)0^+} = +\infty$$

Asymptote verticale double $x = -3$

$$\lim_{x \to \pm\infty} \frac{x+2}{x(x+3)} = \lim_{x \to \pm\infty} \frac{x}{x^2} = \lim_{x \to \pm\infty} \frac{1}{x} = 0$$

Asymptote horizontale double $y = 0$



Corrigé de l'exercice 5

$$D_f = \mathbb{R}$$

Pas d'asymptote verticale.

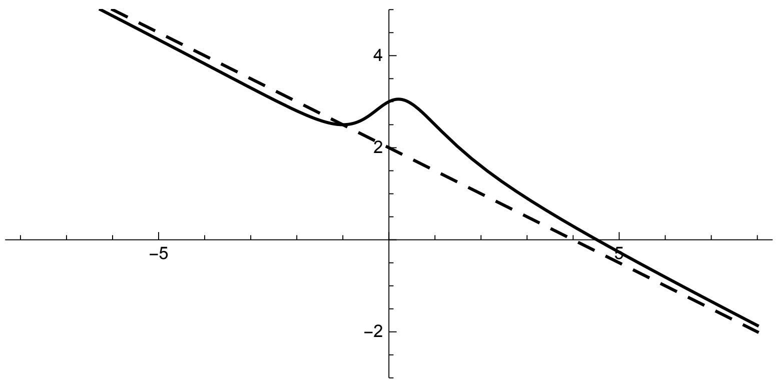
$$f(x) = 2 + \frac{x+1}{x^2+1} - \frac{x}{2} = \left(-\frac{1}{2}x + 2\right) + \frac{x+1}{x^2+1}$$

Asymptote affine double $y = -\frac{1}{2}x + 2$ car

$$e(x) = f(x) - \left(-\frac{1}{2}x + 2\right) = \frac{x+1}{x^2+1} \quad \text{avec}$$

$$\lim_{x \to \pm\infty} e(x) = \lim_{x \to \pm\infty} \frac{x+1}{x^2+1} = \lim_{x \to \pm\infty} \frac{x}{x^2} = \lim_{x \to \pm\infty} \frac{1}{x} = 0$$

x	$-\infty$	-1	∞
$\text{sign}(x+1)$	$-$	0	$+$
$\text{sign}\,(x^2+1)$	$+$	$+$	$+$
$\text{sign}(e(x))$	$-$	0	$+$
Position relative	Asymptote Fonction	Intersection	Fonction Asymptote



Corrigé de l'exercice 6

$$D_f = \mathbb{R}\backslash\{1\}$$

$$f'(x) = \frac{(-2x+1)(x-1) - (-x^2+x-4)\,1}{(x-1)^2} = \frac{-x^2+2x+3}{(x-1)^2}$$

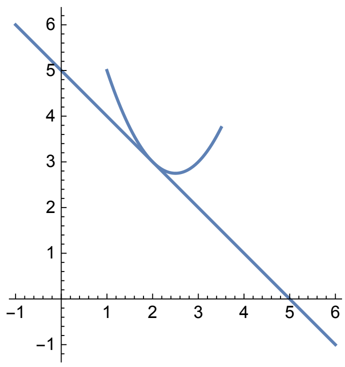
$$Z_{f'} = \{-1; 3\}$$

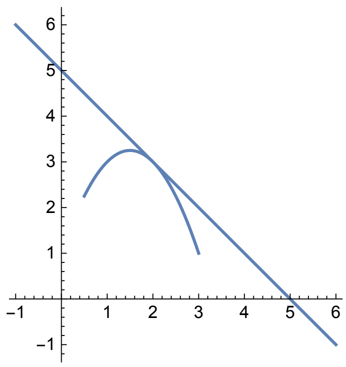
x	$-\infty$	-1		1		3	∞
$\text{sign}(-x^2+2x+3)$	$-$	0	$+$	$+$	$+$	0	$-$
$\text{sign}\left((x-1)^2\right)$	$+$	$+$	$+$	0	$+$	$+$	$+$
$\text{sign}(f'(x))$	$-$	0	$+$	$\Vert$	$+$	0	$-$
	∞		∞	$\Vert$		-5	
$\text{Var}(f(x))$	$\searrow$		$\nearrow$	$\Vert$	$\nearrow$		$\searrow$
		3		$\Vert$	$-\infty$		$-\infty$

$$f(-1) = 3; \text{ minimumen } (-1; 3)$$
$$f(3) = -5; \text{ maximumen } (3; -5)$$

Corrigé de l'exercice 7

Représentons schématiquement quelques situations possibles au voisinage du point de tangence





1-ère condition : la courbe passe par le point $(2; 3)$

$$f(x) = 3 \iff a\,2 + \frac{b}{2} = 3$$

2-ème condition : en $x = 2$, la pente de la courbe est de -1

$$f'(2) = -1 \quad \text{où } f'(x) = a - \frac{b}{x^2}$$

$$a - \frac{b}{2^2} = -1$$

Système

$$\begin{cases} 2a + \frac{1}{2}b = 3 \\ a - \frac{1}{4}b = -1 \end{cases} \iff \begin{cases} 2a + \frac{1}{2}b = 3 \\ 2a - \frac{1}{2}b = -2 \end{cases} \implies 4a = 1$$

$$\implies a = \frac{1}{4} \quad \text{et } b = 2(3 - 2a) = 5$$

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Marcel Délèze