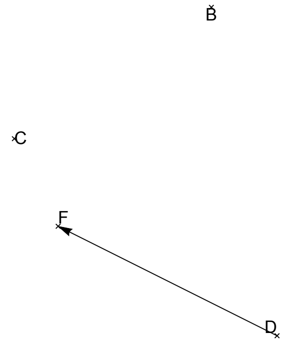
Énoncés des exercices « Géométrie vectorielle dans le plan »

www.deleze.name/marcel/sec2/ex-corriges/1/vecteurs_2d.pdf

Géométrie vectorielle dans le plan - Corrigés

Corrigé de l'exercice 1

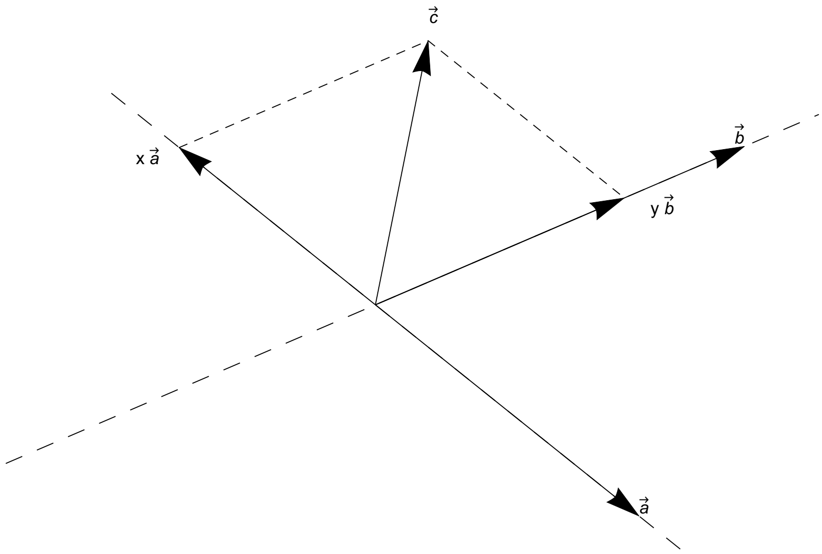


a) $\overrightarrow{CE} = \overrightarrow{AD} - 2\overrightarrow{BC} = \overrightarrow{AD} + 2\overrightarrow{CB}$

b) $\overrightarrow{DF} = \frac{5}{3}\overrightarrow{AB}$

Corrigé de l'exercice 2

a) $\vec{c} = x\,\vec{a} + y\,\vec{b}$



Estimation : $x \approx -\frac{3}{4}$; $y \approx \frac{2}{3}$

b) $x\,\vec{a} + y\,\vec{b} = \vec{c}$

$$x \begin{pmatrix} 5 \\ -4 \end{pmatrix} + y \begin{pmatrix} 7 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$$\left\{ \begin{array}{rrcr|l} 5x & +7y & = & 1 & \cdot 4 \\ -4x & +3y & = & 5 & \cdot 5 \end{array} \right.$$

$$43y = 29;\ y = \frac{29}{43}$$

$$x = \frac{1-7y}{5} = \frac{1-\frac{203}{43}}{5} = \frac{-160}{215} = -\frac{32}{43}$$

$$\vec{c} = -\frac{32}{43}\vec{a} + \frac{29}{43}\vec{b}$$

Corrigé de l'exercice 3

a)

$$\sqrt{(2m-1)^2+4^2} = 7$$
$$(2m-1)^2+16 = 49$$
$$4m^2-4m+1+16-49 = 0$$
$$4m^2-4m-32 = 0$$
$$m^2-m-8 = 0$$

$$\Delta = (-1)^2-4\cdot 1\cdot(-8) = 33; m_1 = \frac{1-\sqrt{33}}{2} \simeq -2.37228;\ m_2 = \frac{1+\sqrt{33}}{2} \simeq 3.37228$$

b)

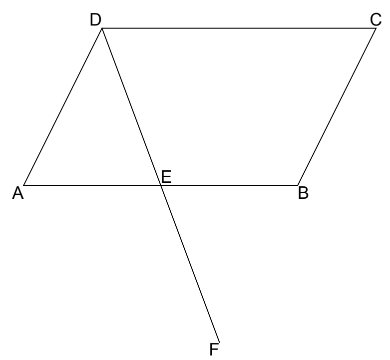
$$\begin{vmatrix} m+1 & 3 \\ 2 & m-1 \end{vmatrix} = (m+1)(m-1)-2\cdot 3 = m^2-1-6 = m^2-7 = 0$$

$$m \in \left\{-\sqrt{7}, \sqrt{7}\right\}$$

c)

$$\begin{pmatrix} 3m \\ 5 \end{pmatrix} \perp \begin{pmatrix} 2 \\ m \end{pmatrix} \iff 3m\cdot 2+5\cdot m = 0 \iff 11m = 0 \iff m = 0$$

Corrigé de l'exercice 4



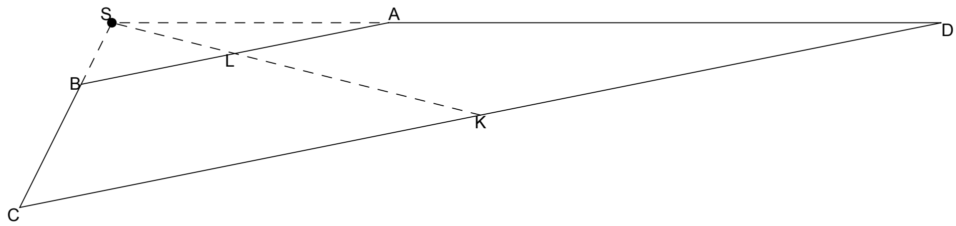
$$\text{Hypothèses : } \overrightarrow{\mathrm{FE}} = \overrightarrow{\mathrm{ED}},\ \overrightarrow{\mathrm{EB}} = \overrightarrow{\mathrm{AE}},\ \overrightarrow{\mathrm{CB}} = \overrightarrow{\mathrm{DA}}.$$

$$\text{Conclusion : } \overrightarrow{\mathrm{FB}} = \overrightarrow{\mathrm{BC}} \iff \overrightarrow{\mathrm{FB}} - \overrightarrow{\mathrm{BC}} = \vec{0} \iff \overrightarrow{\mathrm{FB}} + \overrightarrow{\mathrm{CB}} = \vec{0}.$$

En effet,

$$\begin{aligned}
\overrightarrow{FB} + \overrightarrow{CB} &= \overrightarrow{FE} + \overrightarrow{EB} + \overrightarrow{CB} \\
&= \overrightarrow{ED} + \overrightarrow{EB} + \overrightarrow{DA} \\
&= \overrightarrow{EB} + \overrightarrow{ED} + \overrightarrow{DA} \\
&= \overrightarrow{EB} + \overrightarrow{EA} \\
&= \overrightarrow{AE} + \overrightarrow{EA} \\
&= \overrightarrow{AA} \\
&= \overrightarrow{0} \qquad \blacksquare
\end{aligned}$$

Corrigé de la question 5



a) Par rapport à la base $\left(\overrightarrow{i}, \overrightarrow{j}\right)$

$$\overrightarrow{BA} = \begin{pmatrix} 3-(-2) \\ 5-4 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix};\ \overrightarrow{CD} = \begin{pmatrix} 12-(-3) \\ 5-2 \end{pmatrix} = \begin{pmatrix} 15 \\ 3 \end{pmatrix};\overrightarrow{CD} = 3\overrightarrow{BA}$$

b) Par rapport à la base $\left(\overrightarrow{BA}, \overrightarrow{BC}\right)$

$$\overrightarrow{DA} = \overrightarrow{DC} + \overrightarrow{CB} + \overrightarrow{BA} = -3\overrightarrow{BA} - \overrightarrow{BC} + \overrightarrow{BA} = -2\overrightarrow{BA} - \overrightarrow{BC} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$\begin{aligned}
\overrightarrow{KL} = \overrightarrow{KC} + \overrightarrow{CB} + \overrightarrow{BL} &= -\frac{1}{2}\overrightarrow{CD} - \overrightarrow{BC} + \frac{1}{2}\overrightarrow{BA} \\
&= -\frac{3}{2}\overrightarrow{BA} - \overrightarrow{BC} + \frac{1}{2}\overrightarrow{BA} = -\overrightarrow{BA} - \overrightarrow{BC} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}
\end{aligned}$$

c) Par rapport à la base $\left(\overrightarrow{BA}, \overrightarrow{BC}\right)$

$$\overrightarrow{CS_1} = \overrightarrow{CD} + \overrightarrow{DS_1} = 3\overrightarrow{BA} + \frac{3}{2}\overrightarrow{DA} = 3\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{3}{2}\begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix}$$

$$\overrightarrow{CS_2} = \overrightarrow{CK} + \overrightarrow{KS_2} = \frac{1}{2}\overrightarrow{CD} + \frac{3}{2}\overrightarrow{KL} = \frac{3}{2}\overrightarrow{BA} + \frac{3}{2}\overrightarrow{KL} = \frac{3}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{3}{2}\begin{pmatrix} -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix}$$

$$\overrightarrow{CS_3} = \frac{3}{2}\overrightarrow{CB} = -\frac{3}{2}\overrightarrow{BC} = -\frac{3}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix}$$

Donc

$$\overrightarrow{CS_1} = \overrightarrow{CS_2} = \overrightarrow{CS_3}$$

$$S_1 = S_2 = S_3 = S$$

Les droites AD, KL et BC sont concourantes en S.

Corrigé de l'exercice 6

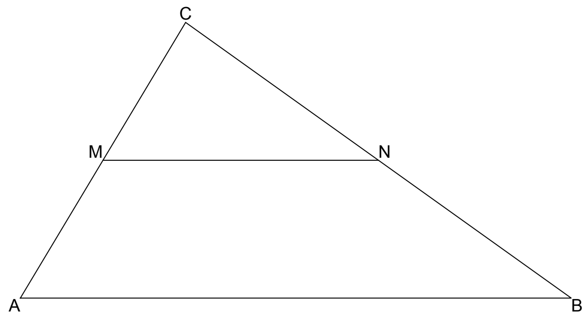
a)

$$\|\vec{a}-\vec{b}\| = \left\| \begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} -4 \\ 3 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 9 \\ -5 \end{pmatrix} \right\| = \sqrt{9^2+(-5)^2} = \sqrt{106}$$

b)

$$\begin{gathered}
\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=\overrightarrow{BC} \\
\overrightarrow{PB}+\overrightarrow{BA}+\overrightarrow{PB}+\overrightarrow{PB}+\overrightarrow{BC}=\overrightarrow{BC} \\
3\overrightarrow{PB}=\overrightarrow{BC}-\overrightarrow{BA}-\overrightarrow{BC} \\
3\overrightarrow{PB}=\overrightarrow{AB} \\
\overrightarrow{PB}=\frac{1}{3}\overrightarrow{AB} \\
\overrightarrow{BP}=\frac{1}{3}\overrightarrow{BA}
\end{gathered}$$

Corrigé de l'exercice 7



$$\overrightarrow{MN}=\overrightarrow{MC}+\overrightarrow{CN}=\frac{1}{2}\overrightarrow{AC}+\frac{1}{2}\overrightarrow{CB}=\frac{1}{2}\left(\overrightarrow{AC}+\overrightarrow{CB}\right)=\frac{1}{2}\overrightarrow{AB} \qquad \blacksquare$$

Corrigé de l'exercice 8

$$\overrightarrow{CN}=\frac{1}{2}\overrightarrow{AC} \quad \text{où} \quad N(x;y)$$

$$\begin{pmatrix} x-4.6 \\ y+1.3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 4.6+2.7 \\ -1.3-3.2 \end{pmatrix}$$

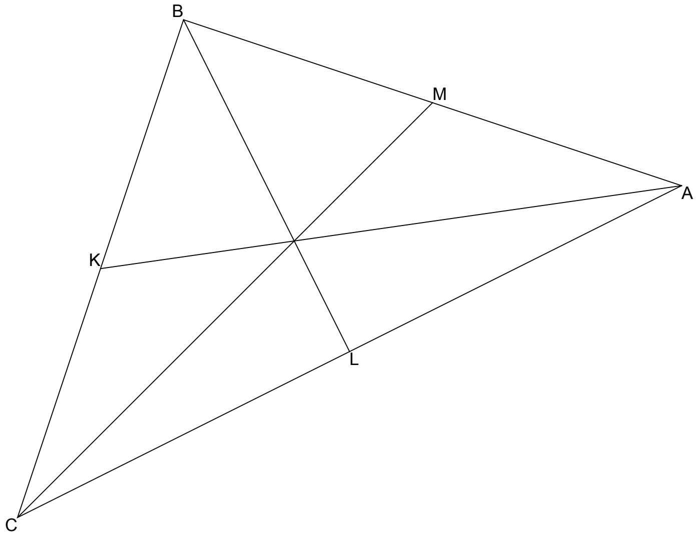
$$\begin{pmatrix} x-4.6 \\ y+1.3 \end{pmatrix} = \begin{pmatrix} 3.65 \\ -2.25 \end{pmatrix}$$

$$x=8.25; \quad y=-3.55; \quad N(8.25;-3.55)=N\left(\frac{33}{4};-\frac{71}{20}\right)$$

Corrigé de l'exercice 9

Hypothèses

$$\overrightarrow{KC} = \frac{1}{2}\overrightarrow{BC};\ \overrightarrow{LA} = \frac{1}{2}\overrightarrow{CA};\ \overrightarrow{MB} = \frac{1}{2}\overrightarrow{AB}$$



Conclusion

$$\overrightarrow{KA} + \overrightarrow{LB} + \overrightarrow{MC} = \vec{0}$$

En effet,

$$\begin{aligned}
\overrightarrow{KA} + \overrightarrow{LB} + \overrightarrow{MC} &= \overrightarrow{KC} + \overrightarrow{CA} + \overrightarrow{LA} + \overrightarrow{AB} + \overrightarrow{MB} + \overrightarrow{BC} \\
&\overset{\text{Hyp}}{=} \frac{1}{2}\overrightarrow{BC} + \overrightarrow{CA} + \frac{1}{2}\overrightarrow{CA} + \overrightarrow{AB} + \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC} \\
&= \frac{3}{2}\overrightarrow{BC} + \frac{3}{2}\overrightarrow{CA} + \frac{3}{2}\overrightarrow{AB} \\
&= \frac{3}{2}\left(\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB}\right) \\
&= \frac{3}{2}\overrightarrow{BB} \\
&= \frac{3}{2}\vec{0} \\
&= \vec{0} \qquad \blacksquare
\end{aligned}$$

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