Eléments finis tétraédriques de classe C^1 et de degré deux
Variante II

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Forme explicite des 52 fonctions de référence

■ Introduction [voir thèse p. 70]

L'élément fini de la Variante II est constitué de fonctions de la forme

$$v = \sum_{i=1}^{28} s_i \tilde{u}_i \circ L^{-1}$$

où les fonctions de pseudo-référence $\tilde{u}_{17}, \ldots, \tilde{u}_{28}$ dépendent de paramètres. Nous reformulons maintenant ces fonctions de manière à les exprimer sous la forme

$$v = \sum_{j=1}^{52} b_j \hat{u}_j \circ L^{-1}$$

où $\hat{u}_1, \ldots, \hat{u}_{52}$ sont des fonctions de référence, c'est-à-dire sont indépendantes du tétraèdre générique. Le but de cet addenda est de définir explicitement ces 52 fonctions de référence.

■ Fonctions de référence 1 à 16

Les 16 première fonctions $\tilde{u}_1, \ldots, \tilde{u}_{16}$ ne dépendant pas de paramètres, elles peuvent être considérées comme des fonctions de référence:

$$\hat{u}_j = \tilde{u}_j \quad \text{pour} \quad j = 1\ (1)\ 16$$

$$b_j = s_j \quad \text{pour} \quad j = 1\ (1)\ 16$$

■ Fonctions de référence 17 à 19 [voir thèse p. 64 et 70]

Sous l'hypothèse $\hat{\mu}_{1\,x} + \hat{\mu}_{1\,y} + \hat{\mu}_{1\,z} = 1$, on a

$$\tilde{u}_{17}\left(\hat{\mu}_{1\,x}, \hat{\mu}_{1\,y}, \hat{\mu}_{1\,z};\ x, y, z\right) = \tilde{w}_{11}\left(\hat{\mu}_{1\,x}, \hat{\mu}_{1\,y}, \hat{\mu}_{1\,z};\ x, y, z\right) =$$

$$r_0(x, y, z) + \hat{\mu}_{1\,x} r_1(x, y, z) + \hat{\mu}_{1\,y} r_2(x, y, z) + \hat{\mu}_{1\,z} r_3(x, y, z) =$$

$$r_0(x, y, z) + \left(1 - \hat{\mu}_{1\,y} - \hat{\mu}_{1\,z}\right) r_1(x, y, z) + \hat{\mu}_{1\,y} r_2(x, y, z) + \hat{\mu}_{1\,z} r_3(x, y, z) =$$

$$(r_0 + r_1)(x, y, z) + \hat{\mu}_{1\,y}(r_2 - r_1)(x, y, z) + \hat{\mu}_{1\,z}(r_3 - r_1)(x, y, z)$$

A partir des fonctions r_0, r_1, r_2, r_3, définissons les fonctions de référence

$$\hat{u}_{17}(x, y, z) = (r_0 + r_1)(x, y, z) = \hat{R}_1(x, y, z)$$

$$\hat{u}_{18}(x, y, z) = (r_2 - r_1)(x, y, z) = \hat{R}_2(x, y, z)$$

$$\hat{u}_{19}(x, y, z) = (r_3 - r_1)(x, y, z) = \hat{R}_3(x, y, z)$$

On a

$$\tilde{u}_{17}\left(\hat{\mu}_{1x}, \hat{\mu}_{1y}, \hat{\mu}_{1z}; \ldots\right) = \hat{u}_{17} + \hat{\mu}_{1y}\hat{u}_{18} + \hat{\mu}_{1z}\hat{u}_{19}$$

Les coefficients qui correspondent aux fonctions de référence peuvent se calculer comme suit

$$s_{17}\tilde{u}_{17}\circ L^{-1} = s_{17}\left(\hat{u}_{17} + \hat{\mu}_{1y}\hat{u}_{18} + \hat{\mu}_{1z}\hat{u}_{19}\right)\circ L^{-1} = \left(b_{17}\hat{u}_{17} + b_{18}\hat{u}_{18} + b_{19}\hat{u}_{19}\right)\circ L^{-1}$$

où on a posé

$$b_{17} = s_{17}\,;\; b_{18} = s_{17}\hat{\mu}_{1y};\; b_{19} = s_{17}\hat{\mu}_{1z}$$

■ Fonctions de référence 20 à 25 [voir thèse p. 61]

$$\tilde{u}_{18}\left(\hat{\mu}_{1x}, \hat{\mu}_{1y}, \hat{\mu}_{1z}; x, y, z\right) =$$
$$\tilde{w}_{12}\left(\hat{\mu}_{1x}, \hat{\mu}_{1y}, \hat{\mu}_{1z}; x, y, z\right) = \tilde{w}_{11}\left(\hat{\mu}_{1y}, \hat{\mu}_{1z}, \hat{\mu}_{1x}; y, z, x\right) =$$
$$(r_0 + r_1)(y, z, x) + \hat{\mu}_{1z}(r_2 - r_1)(y, z, x) + \hat{\mu}_{1x}(r_3 - r_1)(y, z, x)$$

Définissons les fonctions de référence

$$\hat{u}_{20}(x, y, z) = (r_0 + r_1)(y, z, x) = \hat{R}_4(x, y, z)$$

$$\hat{u}_{21}(x, y, z) = (r_2 - r_1)(y, z, x) = \hat{R}_5(x, y, z)$$

$$\hat{u}_{22}(x, y, z) = (r_3 - r_1)(y, z, x) = \hat{R}_6(x, y, z)$$

On a

$$\tilde{u}_{18}\left(\hat{\mu}_{1x}, \hat{\mu}_{1y}, \hat{\mu}_{1z}; \ldots\right) = \hat{u}_{20} + \hat{\mu}_{1z}\hat{u}_{21} + \hat{\mu}_{1x}\hat{u}_{22}$$

Les coefficients qui correspondent aux fonctions de référence se calculent comme suit

$$s_{18}\tilde{u}_{18}\circ L^{-1} = s_{18}\left(\hat{u}_{20} + \hat{\mu}_{1z}\hat{u}_{21} + \hat{\mu}_{1x}\hat{u}_{22}\right)\circ L^{-1} = \left(b_{20}\hat{u}_{20} + b_{21}\hat{u}_{21} + b_{22}\hat{u}_{22}\right)\circ L^{-1}$$

où on a posé

$$b_{20} = s_{18}\,;\; b_{21} = s_{18}\hat{\mu}_{1z};\; b_{22} = s_{18}\hat{\mu}_{1x}$$

$$\tilde{u}_{19}\left(\hat{\mu}_{1x}, \hat{\mu}_{1y}, \hat{\mu}_{1z}; x, y, z\right) =$$
$$\tilde{w}_{13}\left(\hat{\mu}_{1x}, \hat{\mu}_{1y}, \hat{\mu}_{1z}; x, y, z\right) = \tilde{w}_{11}\left(\hat{\mu}_{1z}, \hat{\mu}_{1x}, \hat{\mu}_{1y}; z, x, y\right) =$$
$$(r_0 + r_1)(z, x, y) + \hat{\mu}_{1x}(r_2 - r_1)(z, x, y) + \hat{\mu}_{1y}(r_3 - r_1)(z, x, y)$$

Définissons les fonctions de référence

$$\hat{u}_{23}(x, y, z) = (r_0 + r_1)(z, x, y) = \hat{R}_7(x, y, z)$$

$$\hat{u}_{24}(x, y, z) = (r_2 - r_1)(z, x, y) = \hat{R}_8(x, y, z)$$

$$\hat{u}_{25}(x, y, z) = (r_3 - r_1)(z, x, y) = \hat{R}_9(x, y, z)$$

On a

$$\tilde{u}_{19}\left(\hat{\mu}_{1x}, \hat{\mu}_{1y}, \hat{\mu}_{1z}; \ldots\right) = \hat{u}_{23} + \hat{\mu}_{1x}\hat{u}_{24} + \hat{\mu}_{1y}\hat{u}_{25}$$

Les coefficients qui correspondent aux fonctions de référence se calculent comme suit

$$s_{19}\,\tilde{u}_{19}\circ L^{-1} = s_{19}\left(\hat{u}_{23} + \hat{\mu}_{1\,x}\,\hat{u}_{24} + \hat{\mu}_{1\,y}\,\hat{u}_{25}\right)\circ L^{-1} = \left(b_{23}\,\hat{u}_{23} + b_{24}\,\hat{u}_{24} + b_{25}\,\hat{u}_{25}\right)\circ L^{-1}$$

où on a posé

$$b_{23} = s_{19}\,;\; b_{24} = s_{19}\,\hat{\mu}_{1\,x};\; b_{25} = s_{19}\,\hat{\mu}_{1\,y}$$

Posons pour la suite

$$\tilde{w}_{11}\,(\mu_x,\,\mu_y,\,\mu_z;\;x,\,y,\,z) = \hat{R}_1\,(x,\,y,\,z) + \mu_y\,\hat{R}_2\,(x,\,y,\,z) + \mu_z\,\hat{R}_3\,(x,\,y,\,z)$$

$$\tilde{w}_{12}\,(\mu_x,\,\mu_y,\,\mu_z;\;x,\,y,\,z) = \hat{R}_4\,(x,\,y,\,z) + \mu_z\,\hat{R}_5\,(x,\,y,\,z) + \mu_x\,\hat{R}_6\,(x,\,y,\,z)$$

$$\tilde{w}_{13}\,(\mu_x,\,\mu_y,\,\mu_z;\;x,\,y,\,z) = \hat{R}_7\,(x,\,y,\,z) + \mu_x\,\hat{R}_8\,(x,\,y,\,z) + \mu_y\,\hat{R}_9\,(x,\,y,\,z)$$

■ Fonctions de référence 26 à 34

$$\tilde{u}_{20}\left(\hat{\mu}_{2\,x},\,\hat{\mu}_{2\,y},\,\hat{\mu}_{2\,z};\;x,\,y,\,z\right) =$$
$$\tilde{w}_{21}\left(\hat{\mu}_{2\,x},\,\hat{\mu}_{2\,y},\,\hat{\mu}_{2\,z};\;x,\,y,\,z\right) = \tilde{w}_{11}\left(\hat{\mu}_{2\,x},\,\hat{\mu}_{2\,y},\,\hat{\mu}_{2\,z};\,\alpha_2^{-1}\,(x,\,y,\,z)\right) =$$
$$\hat{R}_1\left(\alpha_2^{-1}\,(x,\,y,\,z)\right) + \hat{\mu}_{2\,y}\,\hat{R}_2\left(\alpha_2^{-1}\,(x,\,y,\,z)\right) + \hat{\mu}_{2\,z}\,\hat{R}_3\left(\alpha_2^{-1}\,(x,\,y,\,z)\right)$$

Définissons les fonctions de référence

$$\hat{u}_{26}\,(x,\,y,\,z) = \hat{R}_1\left(\alpha_2^{-1}\,(x,\,y,\,z)\right) = \hat{R}_1\,(y,\,z,\,1-x-y-z)$$

$$\hat{u}_{27}\,(x,\,y,\,z) = \hat{R}_2\left(\alpha_2^{-1}\,(x,\,y,\,z)\right) = \hat{R}_2\,(y,\,z,\,1-x-y-z)$$

$$\hat{u}_{28}\,(x,\,y,\,z) = \hat{R}_3\left(\alpha_2^{-1}\,(x,\,y,\,z)\right) = \hat{R}_3\,(y,\,z,\,1-x-y-z)$$

On a

$$\tilde{u}_{20}\left(\hat{\mu}_{2\,x},\,\hat{\mu}_{2\,y},\,\hat{\mu}_{2\,z};\;\ldots\right) = \hat{u}_{26} + \hat{\mu}_{2\,y}\,\hat{u}_{27} + \hat{\mu}_{2\,z}\,\hat{u}_{28}$$

Les coefficients qui correspondent aux fonctions de référence se calculent comme suit

$$s_{20}\,\tilde{u}_{20}\circ L^{-1} = s_{20}\left(\hat{u}_{26} + \hat{\mu}_{2\,y}\,\hat{u}_{27} + \hat{\mu}_{2\,z}\,\hat{u}_{28}\right)\circ L^{-1} = \left(b_{26}\,\hat{u}_{26} + b_{27}\,\hat{u}_{27} + b_{28}\,\hat{u}_{28}\right)\circ L^{-1}$$

où on a posé

$$b_{26} = s_{20}\,;\; b_{27} = s_{20}\,\hat{\mu}_{2\,y};\; b_{28} = s_{20}\,\hat{\mu}_{2\,z}$$

$$\tilde{u}_{21}\left(\hat{\mu}_{2\,x},\,\hat{\mu}_{2\,y},\,\hat{\mu}_{2\,z};\;x,\,y,\,z\right) =$$
$$\tilde{w}_{22}\left(\hat{\mu}_{2\,x},\,\hat{\mu}_{2\,y},\,\hat{\mu}_{2\,z};\;x,\,y,\,z\right) = \tilde{w}_{12}\left(\hat{\mu}_{2\,x},\,\hat{\mu}_{2\,y},\,\hat{\mu}_{2\,z};\,\alpha_2^{-1}\,(x,\,y,\,z)\right) =$$
$$\hat{R}_4\left(\alpha_2^{-1}\,(x,\,y,\,z)\right) + \hat{\mu}_{2\,z}\,\hat{R}_5\left(\alpha_2^{-1}\,(x,\,y,\,z)\right) + \hat{\mu}_{2\,x}\,\hat{R}_6\left(\alpha_2^{-1}\,(x,\,y,\,z)\right)$$

Définissons les fonctions de référence

$$\hat{u}_{29}\,(x,\,y,\,z) = \hat{R}_4\left(\alpha_2^{-1}\,(x,\,y,\,z)\right) = \hat{R}_4\,(y,\,z,\,1-x-y-z)$$

$$\hat{u}_{30}\,(x,\,y,\,z) = \hat{R}_5\left(\alpha_2^{-1}\,(x,\,y,\,z)\right) = \hat{R}_5\,(y,\,z,\,1-x-y-z)$$

$$\hat{u}_{31}\,(x,\,y,\,z) = \hat{R}_6\left(\alpha_2^{-1}\,(x,\,y,\,z)\right) = \hat{R}_6\,(y,\,z,\,1-x-y-z)$$

On a

$$\tilde{u}_{21}\left(\hat{\mu}_{2\,x},\ \hat{\mu}_{2\,y},\ \hat{\mu}_{2\,z};\ \ldots\right) = \hat{u}_{29} + \hat{\mu}_{2\,z}\,\hat{u}_{30} + \hat{\mu}_{2\,x}\,\hat{u}_{31}$$

Les coefficients qui correspondent aux fonctions de référence se calculent comme suit

$$s_{21}\,\tilde{u}_{21}\circ L^{-1} = s_{21}\left(\hat{u}_{29} + \hat{\mu}_{2\,z}\,\hat{u}_{30} + \hat{\mu}_{2\,x}\,\hat{u}_{31}\right)\circ L^{-1} = \left(b_{29}\,\hat{u}_{29} + b_{30}\,\hat{u}_{30} + b_{31}\,\hat{u}_{31}\right)\circ L^{-1}$$

où on a posé

$$b_{29} = s_{21}\ ;\ b_{30} = s_{21}\,\hat{\mu}_{2\,z};\ b_{31} = s_{21}\,\hat{\mu}_{2\,x}$$

$$\tilde{u}_{22}\left(\hat{\mu}_{2\,x},\ \hat{\mu}_{2\,y},\ \hat{\mu}_{2\,z};\ x,\ y,\ z\right) =$$
$$\tilde{w}_{23}\left(\hat{\mu}_{2\,x},\ \hat{\mu}_{2\,y},\ \hat{\mu}_{2\,z};\ x,\ y,\ z\right) = \tilde{w}_{13}\left(\hat{\mu}_{2\,x},\ \hat{\mu}_{2\,y},\ \hat{\mu}_{2\,z};\ \alpha_2^{-1}(x,\ y,\ z)\right) =$$
$$\hat{R}_7\left(\alpha_2^{-1}(x,\ y,\ z)\right) + \hat{\mu}_{2\,x}\ \hat{R}_8\left(\alpha_2^{-1}(x,\ y,\ z)\right) + \hat{\mu}_{2\,y}\ \hat{R}_9\left(\alpha_2^{-1}(x,\ y,\ z)\right)$$

Définissons les fonctions de référence

$$\hat{u}_{32}(x,\ y,\ z) = \hat{R}_7\left(\alpha_2^{-1}(x,\ y,\ z)\right) = \hat{R}_7(y,\ z,\ 1-x-y-z)$$

$$\hat{u}_{33}(x,\ y,\ z) = \hat{R}_8\left(\alpha_2^{-1}(x,\ y,\ z)\right) = \hat{R}_8(y,\ z,\ 1-x-y-z)$$

$$\hat{u}_{34}(x,\ y,\ z) = \hat{R}_9\left(\alpha_2^{-1}(x,\ y,\ z)\right) = \hat{R}_9(y,\ z,\ 1-x-y-z)$$

On a

$$\tilde{u}_{22}\left(\hat{\mu}_{2\,x},\ \hat{\mu}_{2\,y},\ \hat{\mu}_{2\,z};\ \ldots\right) = \hat{u}_{32} + \hat{\mu}_{2\,x}\,\hat{u}_{33} + \hat{\mu}_{2\,y}\,\hat{u}_{34}$$

Les coefficients qui correspondent aux fonctions de référence se calculent comme suit

$$s_{22}\,\tilde{u}_{22}\circ L^{-1} = s_{22}\left(\hat{u}_{32} + \hat{\mu}_{2\,x}\,\hat{u}_{33} + \hat{\mu}_{2\,y}\,\hat{u}_{34}\right)\circ L^{-1} = \left(b_{32}\,\hat{u}_{32} + b_{33}\,\hat{u}_{33} + b_{34}\,\hat{u}_{34}\right)\circ L^{-1}$$

où on a posé

$$b_{32} = s_{22}\ ;\ b_{33} = s_{22}\,\hat{\mu}_{2\,x};\ b_{34} = s_{22}\,\hat{\mu}_{2\,y}$$

■ Fonctions de référence 35 à 43

$$\tilde{u}_{23}\left(\hat{\mu}_{3\,x},\ \hat{\mu}_{3\,y},\ \hat{\mu}_{3\,z};\ x,\ y,\ z\right) =$$
$$\tilde{w}_{31}\left(\hat{\mu}_{3\,x},\ \hat{\mu}_{3\,y},\ \hat{\mu}_{3\,z};\ x,\ y,\ z\right) = \tilde{w}_{11}\left(\hat{\mu}_{3\,x},\ \hat{\mu}_{3\,y},\ \hat{\mu}_{3\,z};\ \alpha_3^{-1}(x,\ y,\ z)\right)\Big) =$$
$$\hat{R}_1\left(\alpha_3^{-1}(x,\ y,\ z)\right) + \hat{\mu}_{3\,y}\ \hat{R}_2\left(\alpha_3^{-1}(x,\ y,\ z)\right) + \hat{\mu}_{3\,z}\ \hat{R}_3\left(\alpha_3^{-1}(x,\ y,\ z)\right)$$

Définissons les fonctions de référence

$$\hat{u}_{35}(x,\ y,\ z) = \hat{R}_1\left(\alpha_3^{-1}(x,\ y,\ z)\right) = \hat{R}_1(z,\ 1-x-y-z,\ x)$$

$$\hat{u}_{36}(x,\ y,\ z) = \hat{R}_2\left(\alpha_3^{-1}(x,\ y,\ z)\right) = \hat{R}_2(z,\ 1-x-y-z,\ x)$$

$$\hat{u}_{37}(x,\ y,\ z) = \hat{R}_2\left(\alpha_3^{-1}(x,\ y,\ z)\right) = \hat{R}_3(z,\ 1-x-y-z,\ x)$$

On a

$$\tilde{u}_{23}\left(\hat{\mu}_{3\,x},\ \hat{\mu}_{3\,y},\ \hat{\mu}_{3\,z};\ \ldots\right) = \hat{u}_{35} + \hat{\mu}_{3\,y}\,\hat{u}_{36} + \hat{\mu}_{3\,z}\ \hat{u}_{37}$$

Les coefficients qui correspondent aux fonctions de référence peuvent se calculer comme suit

$$s_{23}\,\tilde{u}_{23}\circ L^{-1} = s_{23}\left(\hat{u}_{35} + \hat{\mu}_{3\,y}\,\hat{u}_{36} + \hat{\mu}_{3\,z}\,\hat{u}_{37}\right)\circ L^{-1} = \left(b_{35}\,\hat{u}_{35} + b_{36}\,\hat{u}_{36} + b_{37}\,\hat{u}_{37}\right)\circ L^{-1}$$

où on a posé

$$b_{35} = s_{23}\,;\; b_{36} = s_{23}\,\hat{\mu}_{3\,y};\; b_{37} = s_{23}\,\hat{\mu}_{3\,z}$$

$$\tilde{u}_{24}\left(\hat{\mu}_{3\,x}, \hat{\mu}_{3\,y}, \hat{\mu}_{3\,z};\, x, y, z\right) =$$
$$\tilde{w}_{32}\left(\hat{\mu}_{3\,x}, \hat{\mu}_{3\,y}, \hat{\mu}_{3\,z};\, x, y, z\right) = \tilde{w}_{12}\left(\hat{\mu}_{3\,x}, \hat{\mu}_{3\,y}, \hat{\mu}_{3\,z};\, \alpha_3^{-1}(x, y, z)\right)\Big) =$$
$$\hat{R}_4\left(\alpha_3^{-1}(x, y, z)\right) + \hat{\mu}_{3\,z}\,\hat{R}_5\left(\alpha_3^{-1}(x, y, z)\right) + \hat{\mu}_{3\,x}\,\hat{R}_6\left(\alpha_3^{-1}(x, y, z)\right)$$

Définissons les fonctions de référence

$$\hat{u}_{38}(x, y, z) = \hat{R}_4\left(\alpha_3^{-1}(x, y, z)\right) = \hat{R}_4(z, 1 - x - y - z, x)$$

$$\hat{u}_{39}(x, y, z) = \hat{R}_5\left(\alpha_3^{-1}(x, y, z)\right) = \hat{R}_5(z, 1 - x - y - z, x)$$

$$\hat{u}_{40}(x, y, z) = \hat{R}_6\left(\alpha_3^{-1}(x, y, z)\right) = \hat{R}_6(z, 1 - x - y - z, x)$$

On a

$$\tilde{u}_{24}\left(\hat{\mu}_{3\,x}, \hat{\mu}_{3\,y}, \hat{\mu}_{3\,z};\, \ldots\right) = \hat{u}_{38} + \hat{\mu}_{3\,z}\,\hat{u}_{39} + \hat{\mu}_{3\,x}\,\hat{u}_{40}$$

Les coefficients qui correspondent aux fonctions de référence se calculent comme suit

$$s_{24}\,\tilde{u}_{24}\circ L^{-1} = s_{24}\left(\hat{u}_{38} + \hat{\mu}_{3\,z}\,\hat{u}_{39} + \hat{\mu}_{3\,x}\,\hat{u}_{40}\right)\circ L^{-1} = \left(b_{38}\,\hat{u}_{38} + b_{39}\,\hat{u}_{39} + b_{40}\,\hat{u}_{40}\right)\circ L^{-1}$$

où on a posé

$$b_{38} = s_{24}\,;\; b_{39} = s_{24}\,\hat{\mu}_{3\,z}\,;\; b_{40} = s_{24}\,\hat{\mu}_{3\,x}$$

$$\tilde{u}_{25}\left(\hat{\mu}_{3\,x}, \hat{\mu}_{3\,y}, \hat{\mu}_{3\,z};\, x, y, z\right) =$$
$$\tilde{w}_{33}\left(\hat{\mu}_{3\,x}, \hat{\mu}_{3\,y}, \hat{\mu}_{3\,z};\, x, y, z\right) = \tilde{w}_{13}\left(\hat{\mu}_{3\,x}, \hat{\mu}_{3\,y}, \hat{\mu}_{3\,z};\, \alpha_3^{-1}(x, y, z)\right) =$$
$$\hat{R}_7\left(\alpha_3^{-1}(x, y, z)\right) + \hat{\mu}_{3\,x}\,\hat{R}_8\left(\alpha_3^{-1}(x, y, z)\right) + \hat{\mu}_{3\,y}\,\hat{R}_9\left(\alpha_3^{-1}(x, y, z)\right)$$

Définissons les fonctions de référence

$$\hat{u}_{41}(x, y, z) = \hat{R}_7\left(\alpha_3^{-1}(x, y, z)\right) = \hat{R}_7(z, 1 - x - y - z, x)$$

$$\hat{u}_{42}(x, y, z) = \hat{R}_8\left(\alpha_3^{-1}(x, y, z)\right) = \hat{R}_8(z, 1 - x - y - z, x)$$

$$\hat{u}_{43}(x, y, z) = \hat{R}_9\left(\alpha_3^{-1}(x, y, z)\right) = \hat{R}_9(z, 1 - x - y - z, x)$$

On a

$$\tilde{u}_{25}\left(\hat{\mu}_{3\,x}, \hat{\mu}_{3\,y}, \hat{\mu}_{3\,z};\, \ldots\right) = \hat{u}_{41} + \hat{\mu}_{3\,x}\,\hat{u}_{42} + \hat{\mu}_{3\,y}\,\hat{u}_{43}$$

Les coefficients qui correspondent aux fonctions de référence se calculent comme suit

$$s_{25}\,\tilde{u}_{25}\circ L^{-1} = s_{25}\left(\hat{u}_{41} + \hat{\mu}_{3\,x}\,\hat{u}_{42} + \hat{\mu}_{3\,y}\,\hat{u}_{43}\right)\circ L^{-1} = \left(b_{41}\,\hat{u}_{41} + b_{42}\,\hat{u}_{42} + b_{43}\,\hat{u}_{43}\right)\circ L^{-1}$$

où on a posé

$$b_{41} = s_{25}\,;\; b_{42} = s_{25}\,\hat{\mu}_{3\,x};\; b_{43} = s_{25}\,\hat{\mu}_{3\,y}$$

Fonctions de référence 44 à 52

$$\tilde{u}_{26}\left(\hat{\mu}_{4\,x},\ \hat{\mu}_{4\,y},\ \hat{\mu}_{4\,z};\ x,\ y,\ z\right) =$$
$$\tilde{w}_{41}\left(\hat{\mu}_{4\,x},\ \hat{\mu}_{4\,y},\ \hat{\mu}_{4\,z};\ x,\ y,\ z\right) = \tilde{w}_{11}\left(\hat{\mu}_{4\,x},\ \hat{\mu}_{4\,y},\ \hat{\mu}_{4\,z};\ \alpha_4^{-1}\,(x,\ y,\ z)\right) =$$
$$\hat{R}_1\left(\alpha_4^{-1}\,(x,\ y,\ z)\right) + \hat{\mu}_{4\,y}\,\hat{R}_2\left(\alpha_4^{-1}\,(x,\ y,\ z)\right) + \hat{\mu}_{4\,z}\,\hat{R}_3\left(\alpha_4^{-1}\,(x,\ y,\ z)\right)$$

Définissons les fonctions de référence

$$\hat{u}_{44}\,(x,\ y,\ z) = \hat{R}_1\left(\alpha_4^{-1}\,(x,\ y,\ z)\right) = \hat{R}_1\,(1 - x - y - z,\ x,\ y)$$
$$\hat{u}_{45}\,(x,\ y,\ z) = \hat{R}_2\left(\alpha_4^{-1}\,(x,\ y,\ z)\right) = \hat{R}_2\,(1 - x - y - z,\ x,\ y)$$
$$\hat{u}_{46}\,(x,\ y,\ z) = \hat{R}_3\left(\alpha_4^{-1}\,(x,\ y,\ z)\right) = \hat{R}_3\,(1 - x - y - z,\ x,\ y)$$

On a

$$\tilde{u}_{26}\left(\hat{\mu}_{4\,x},\ \hat{\mu}_{4\,y},\ \hat{\mu}_{4\,z};\ \ldots\right) = \hat{u}_{44} + \hat{\mu}_{4\,y}\,\hat{u}_{45} + \hat{\mu}_{4\,z}\,\hat{u}_{46}$$

Les coefficients qui correspondent aux fonctions de référence se calculent comme suit

$$s_{26}\,\tilde{u}_{26}\circ L^{-1} = s_{26}\left(\hat{u}_{44} + \hat{\mu}_{4\,y}\,\hat{u}_{45} + \hat{\mu}_{4\,z}\,\hat{u}_{46}\right)\circ L^{-1} = \left(b_{44}\,\hat{u}_{44} + b_{45}\,\hat{u}_{45} + b_{46}\,\hat{u}_{46}\right)\circ L^{-1}$$

où on a posé

$$b_{44} = s_{26}\ ;\ b_{45} = s_{26}\,\hat{\mu}_{4\,y};\ b_{46} = s_{26}\,\hat{\mu}_{4\,z}$$

$$\tilde{u}_{27}\left(\hat{\mu}_{4\,x},\ \hat{\mu}_{4\,y},\ \hat{\mu}_{4\,z};\ x,\ y,\ z\right) =$$
$$\tilde{w}_{42}\left(\hat{\mu}_{4\,x},\ \hat{\mu}_{4\,y},\ \hat{\mu}_{4\,z};\ x,\ y,\ z\right) = \tilde{w}_{12}\left(\hat{\mu}_{4\,x},\ \hat{\mu}_{4\,y},\ \hat{\mu}_{4\,z};\ \alpha_4^{-1}\,(x,\ y,\ z)\right) =$$
$$\hat{R}_4\left(\alpha_4^{-1}\,(x,\ y,\ z)\right) + \hat{\mu}_{4\,z}\,\hat{R}_5\left(\alpha_4^{-1}\,(x,\ y,\ z)\right) + \hat{\mu}_{4\,x}\,\hat{R}_6\left(\alpha_4^{-1}\,(x,\ y,\ z)\right)$$

Définissons les fonctions de référence

$$\hat{u}_{47}\,(x,\ y,\ z) = \hat{R}_4\left(\alpha_4^{-1}\,(x,\ y,\ z)\right) = \hat{R}_4\,(1 - x - y - z,\ x,\ y)$$
$$\hat{u}_{48}\,(x,\ y,\ z) = \hat{R}_5\left(\alpha_4^{-1}\,(x,\ y,\ z)\right) = \hat{R}_5\,(1 - x - y - z,\ x,\ y)$$
$$\hat{u}_{49}\,(x,\ y,\ z) = \hat{R}_6\left(\alpha_4^{-1}\,(x,\ y,\ z)\right) = \hat{R}_6\,(1 - x - y - z,\ x,\ y)$$

On a

$$\tilde{u}_{27}\left(\hat{\mu}_{4\,x},\ \hat{\mu}_{4\,y},\ \hat{\mu}_{4\,z};\ \ldots\right) = \hat{u}_{47} + \hat{\mu}_{4\,z}\,\hat{u}_{48} + \hat{\mu}_{4\,x}\,\hat{u}_{49}$$

Les coefficients qui correspondent aux fonctions de référence se calculent comme suit

$$s_{27}\,\tilde{u}_{21}\circ L^{-1} = s_{27}\left(\hat{u}_{47} + \hat{\mu}_{4\,z}\,\hat{u}_{48} + \hat{\mu}_{4\,x}\,\hat{u}_{49}\right)\circ L^{-1} = \left(b_{47}\,\hat{u}_{47} + b_{48}\,\hat{u}_{48} + b_{49}\,\hat{u}_{49}\right)\circ L^{-1}$$

où on a posé

$$b_{47} = s_{27}\ ;\ b_{48} = s_{27}\,\hat{\mu}_{4\,z};\ b_{49} = s_{27}\,\hat{\mu}_{4\,x}$$

$$\tilde{u}_{28}\left(\hat{\mu}_{4\,x},\ \hat{\mu}_{4\,y},\ \hat{\mu}_{4\,z};\ x,\ y,\ z\right) =$$
$$\tilde{w}_{43}\left(\hat{\mu}_{4\,x},\ \hat{\mu}_{4\,y},\ \hat{\mu}_{4\,z};\ x,\ y,\ z\right) = \tilde{w}_{13}\left(\hat{\mu}_{4\,x},\ \hat{\mu}_{4\,y},\ \hat{\mu}_{4\,z};\ \alpha_4^{-1}\,(x,\ y,\ z)\right) =$$
$$\hat{R}_7\ \left(\alpha_4^{-1}\,(x,\ y,\ z)\right) + \hat{\mu}_{4\,x}\ \hat{R}_8\ \left(\alpha_4^{-1}\,(x,\ y,\ z)\right) + \hat{\mu}_{4\,y}\ \hat{R}_9\ \left(\alpha_4^{-1}\,(x,\ y,\ z)\right)$$

Définissons les fonctions de référence

$$\hat{u}_{50}\,(x,\ y,\ z) = \hat{R}_7\ \left(\alpha_4^{-1}\,(x,\ y,\ z)\right) = \hat{R}_7\ (1 - x - y - z,\ x,\ y)$$

$$\hat{u}_{51}\,(x,\ y,\ z) = \hat{R}_8\ \left(\alpha_4^{-1}\,(x,\ y,\ z)\right) = \hat{R}_8\ (1 - x - y - z,\ x,\ y)$$

$$\hat{u}_{52}\,(x,\ y,\ z) = \hat{R}_9\ \left(\alpha_4^{-1}\,(x,\ y,\ z)\right) = \hat{R}_9\ (1 - x - y - z,\ x,\ y)$$

On a

$$\tilde{u}_{28}\left(\hat{\mu}_{4\,x},\ \hat{\mu}_{4\,y},\ \hat{\mu}_{4\,z};\ \ldots\right) = \hat{u}_{50} + \hat{\mu}_{4\,x}\,\hat{u}_{51} + \hat{\mu}_{4\,y}\,\hat{u}_{52}$$

Les coefficients qui correspondent aux fonctions de référence se calculent comme suit

$$s_{28}\,\tilde{u}_{22} \circ L^{-1} = s_{28}\left(\hat{u}_{50} + \hat{\mu}_{4\,x}\,\hat{u}_{51} + \hat{\mu}_{4\,y}\,\hat{u}_{52}\right) \circ L^{-1} = \left(b_{50}\,\hat{u}_{50} + b_{51}\,\hat{u}_{51} + b_{52}\,\hat{u}_{52}\right) \circ L^{-1}$$

où on a posé

$$b_{50} = s_{28}\ ;\ b_{51} = s_{28}\,\hat{\mu}_{4\,x};\ b_{52} = s_{28}\,\hat{\mu}_{4\,y}$$

Variante II
Expression des 52 fonctions de référence en langage *Mathematica*

- **Fonctions réciproques de α_i, i=1,2,3,4, notées invα[i]   [thèse p. 50]**

```
invα[1][{x_, y_, z_}] := {x, y, z}

invα[2][{x_, y_, z_}] := {y, z, 1 - x - y - z}

invα[3][{x_, y_, z_}] := {z, 1 - x - y - z, x}

invα[4][{x_, y_, z_}] := {1 - x - y - z, x, y}
```

- **Fonctions de référence 1 à 16, polynômes**

$\hat{u}_j$ étant noté uc[j], on a

```
pt[1, 0][{x_, y_, z_}] := (1 - x - y - z) (1 + x + y + z - 2 (x^2 + y^2 + z^2 + x y + y z + z x))

pt[1, 1][{x_, y_, z_}] := x (1 - x - y - z) (1 - x - 1/2 (y + z))

pt[1, 2][{x_, y_, z_}] := pt[1, 1][{y, z, x}]

pt[1, 3][{x_, y_, z_}] := pt[1, 1][{z, x, y}]

pt[i_, j_][{x_, y_, z_}] := pt[1, j][invα[i][{x, y, z}]]

Do[Do[uc[4 i + j - 3] = pt[i, j], {j, 0, 3}], {i, 1, 4}]
```

■ Fonctions de référence 17 à 52, fractions rationnelles:

$\hat{R}_j$ étant noté rc[j], à partir des définitions, thèse p. 64,

$$\text{r0}[\{x\_, y\_, z\_\}] := \frac{2\,x\,y^2\,z^2}{(x+y)\,(x+z)}$$

$$\text{r}[\{x\_, y\_, z\_\}] := \frac{2\,x^2\,y^2\,z^2\,(1-x-y-z)}{(1-x-y)\,(1-x-z)\,(1-y-z)\,(1-y)\,(1-z)}$$

$$\text{r1}[\{x\_, y\_, z\_\}] := \text{r}[\{x, y, z\}]\left(-1-\frac{y\,z\,(1+x)}{(1-y)\,(1-z)}\right)$$

$$\text{r2}[\{x\_, y\_, z\_\}] := \text{r}[\{x, y, z\}]\,\frac{z\,(2\,x+y)}{(1-z)}$$

$$\text{r3}[\{x\_, y\_, z\_\}] := \text{r}[\{x, y, z\}]\,\frac{y\,(2\,x+z)}{(1-y)}$$

$$\hat{R}_1\,(x, y, z) = (r_0 + r_1)\,(x, y, z)$$

$$\textbf{rc[1][\{x\_, y\_, z\_\}]} := \frac{2\,x\,y^2\,z^2}{(x+y)\,(x+z)} + \frac{2\,x^2\,y^2\,z^2\,(1-x-y-z)}{(1-x-y)\,(1-x-z)\,(1-y-z)\,(1-y)\,(1-z)}\left(-1-\frac{y\,z\,(1+x)}{(1-y)\,(1-z)}\right)$$

$$\hat{R}_2\,(x, y, z) = (r_2 - r_1)\,(x, y, z)$$

$$\textbf{rc[2][\{x\_, y\_, z\_\}]} := \frac{2\,x^2\,y^2\,z^2\,(1-x-y-z)}{(1-x-y)\,(1-x-z)\,(1-y-z)\,(1-y)\,(1-z)}\left(1+\frac{(2-y)\,(x+y)\,z}{(1-y)\,(1-z)}\right)$$

$$\hat{R}_2\,(x, y, z) = (r_3 - r_1)\,(x, y, z)$$

$$\textbf{rc[3][\{x\_, y\_, z\_\}]} := \frac{2\,x^2\,y^2\,z^2\,(1-x-y-z)}{(1-x-y)\,(1-x-z)\,(1-y-z)\,(1-y)\,(1-z)}\left(1+\frac{(2-z)\,y\,(x+z)}{(1-y)\,(1-z)}\right)$$

$$\hat{R}_4\,(x, y, z) = (r_0 + r_1)\,(y, z, x) = \hat{R}_1\,(y, z, x)$$

rc[4][{x_, y_, z_}] := rc[1][{y, z, x}]

$$\hat{R}_5\,(x, y, z) = (r_2 - r_1)\,(y, z, x) = \hat{R}_2\,(y, z, x)$$

rc[5][{x_, y_, z_}] := rc[2][{y, z, x}]

$$\hat{R}_6\,(x, y, z) = (r_3 - r_1)\,(y, z, x) = \hat{R}_3\,(y, z, x)$$

rc[6][{x_, y_, z_}] := rc[3][{y, z, x}]

$$\hat{R}_7\,(x, y, z) = (r_0 + r_1)\,(z, x, y) = \hat{R}_1\,(z, x, y)$$

rc[7][{x_, y_, z_}] := rc[1][{z, x, y}]

$$\hat{R}_8\,(x, y, z) = (r_2 - r_1)\,(z, x, y) = \hat{R}_2\,(z, x, y)$$

rc[8][{x_, y_, z_}] := rc[2][{z, x, y}]

$$\hat{R}_9\,(x, y, z) = (r_3 - r_1)\,(z, x, y) = \hat{R}_3\,(z, x, y)$$

rc[9][{x_, y_, z_}] := rc[3][{z, x, y}]

```
Do[uc[16 + j] = rc[j], {j, 1, 9}]

uc[26][{x_, y_, z_}] := rc[1][{y, z, 1 - x - y - z}]

uc[27][{x_, y_, z_}] := rc[2][{y, z, 1 - x - y - z}]

uc[28][{x_, y_, z_}] := rc[3][{y, z, 1 - x - y - z}]

uc[29][{x_, y_, z_}] := rc[4][{y, z, 1 - x - y - z}]

uc[30][{x_, y_, z_}] := rc[5][{y, z, 1 - x - y - z}]

uc[31][{x_, y_, z_}] := rc[6][{y, z, 1 - x - y - z}]

uc[32][{x_, y_, z_}] := rc[7][{y, z, 1 - x - y - z}]

uc[33][{x_, y_, z_}] := rc[8][{y, z, 1 - x - y - z}]

uc[34][{x_, y_, z_}] := rc[9][{y, z, 1 - x - y - z}]

uc[35][{x_, y_, z_}] := rc[1][{z, 1 - x - y - z, x}]

uc[36][{x_, y_, z_}] := rc[2][{z, 1 - x - y - z, x}]

uc[37][{x_, y_, z_}] := rc[3][{z, 1 - x - y - z, x}]

uc[38][{x_, y_, z_}] := rc[4][{z, 1 - x - y - z, x}]

uc[39][{x_, y_, z_}] := rc[5][{z, 1 - x - y - z, x}]

uc[40][{x_, y_, z_}] := rc[6][{z, 1 - x - y - z, x}]

uc[41][{x_, y_, z_}] := rc[7][{z, 1 - x - y - z, x}]

uc[42][{x_, y_, z_}] := rc[8][{z, 1 - x - y - z, x}]

uc[43][{x_, y_, z_}] := rc[9][{z, 1 - x - y - z, x}]

uc[44][{x_, y_, z_}] := rc[1][{1 - x - y - z, x, y}]

uc[45][{x_, y_, z_}] := rc[2][{1 - x - y - z, x, y}]

uc[46][{x_, y_, z_}] := rc[3][{1 - x - y - z, x, y}]

uc[47][{x_, y_, z_}] := rc[4][{1 - x - y - z, x, y}]

uc[48][{x_, y_, z_}] := rc[5][{1 - x - y - z, x, y}]

uc[49][{x_, y_, z_}] := rc[6][{1 - x - y - z, x, y}]

uc[50][{x_, y_, z_}] := rc[7][{1 - x - y - z, x, y}]

uc[51][{x_, y_, z_}] := rc[8][{1 - x - y - z, x, y}]

uc[52][{x_, y_, z_}] := rc[9][{1 - x - y - z, x, y}]
```