Marcel Délèze
Edition 2017

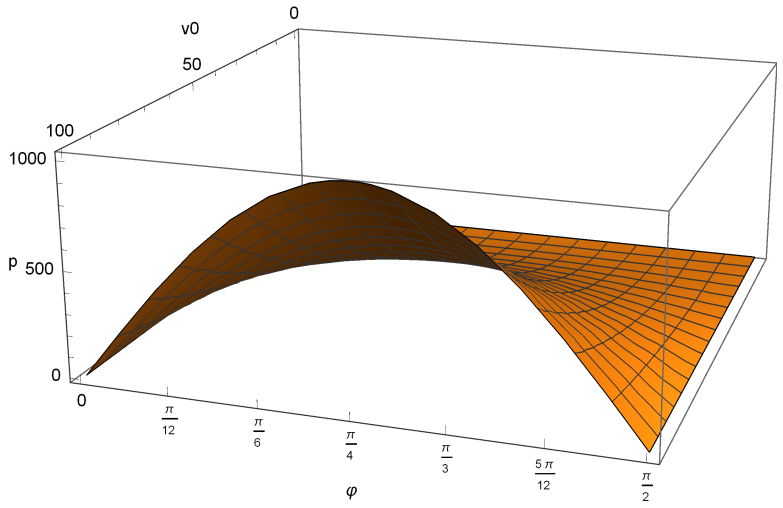
Thème: Fonctions de plusieurs variables § 1.1, 1.2 et 1.3

Lien vers les énoncés des exercices:

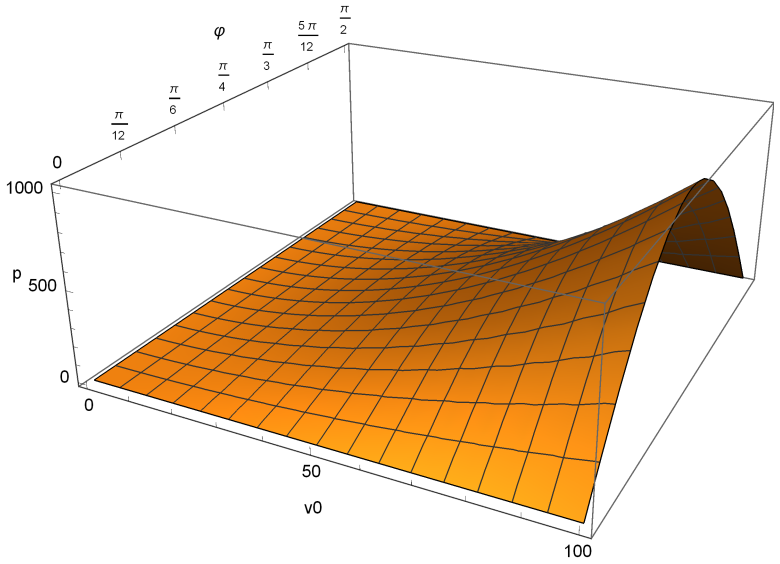
https://www.deleze.name/marcel/sec2/applmaths/csud/plusieurs-variables/1-3_DERIVEES_PARTIELLES.pdf

Corrigé de l'exercice 1-1 a)

```
Plot3D[v0^2 Sin[2 φ]/9.81`, {v0, 0, 100}, {φ, 0, π/2},
 ViewPoint → {3, 1, 1}, AxesLabel → {"v0", "φ", "p"},
 ImageSize → {400, 400}, Ticks → {Automatic, Range[0, π/2, π/12], Automatic}]
```

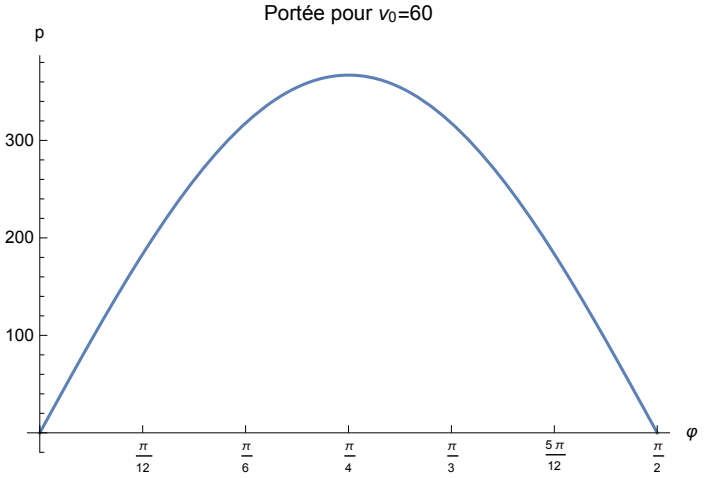


```
Plot3D[v0^2 Sin[2 φ]/9.81`, {v0, 0, 100}, {φ, 0, π/2},
 tracé de surfaces
 ViewPoint → {1, -2, 1}, AxesLabel → {"v0", "φ", "p"},
 point de vue spatial   titre d'axe
 ImageSize → {400, 400}, Ticks → {Automatic, Range[0, π/2, π/12], Automatic}]
 taille d'image   graduati···  automatique  plage  automatique
```

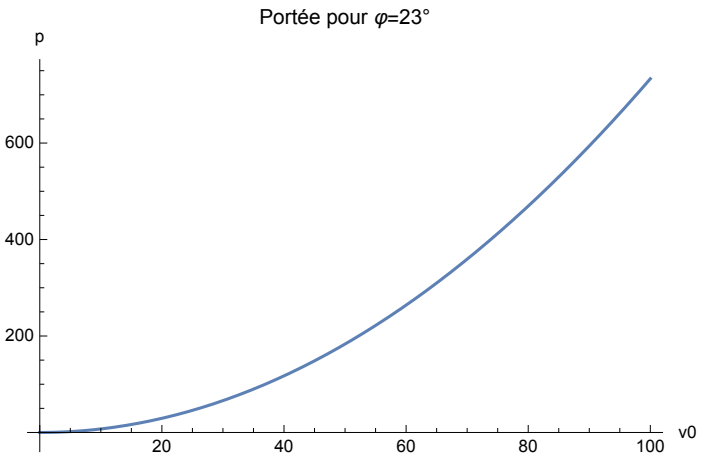


Corrigé de l'exercice 1-1 b)

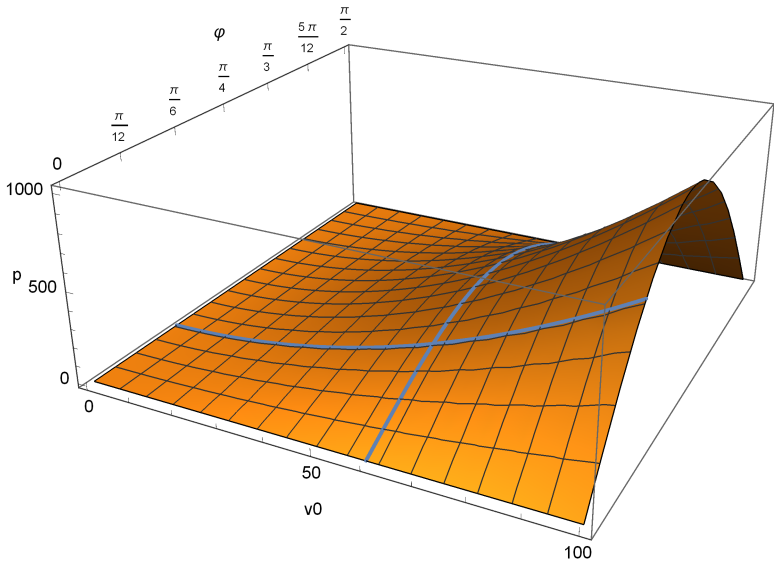
```
Plot[v0^2 Sin[2 φ]/9.81` /. v0 → 60, {φ, 0, π/2}, Ticks → {Range[0, π/2, π/12], Automatic},
 PlotLabel → "Portée pour \!\(\*SubscriptBox[\(v\), \(0\)]\)=60",
 AxesLabel → {"φ", "p"}]
```




```
Plot[v0^2 Sin[2 φ]/9.81` /. φ → 23 °, {v0, 0, 100},
 PlotLabel → "Portée pour φ=23°", AxesLabel → {"v0", "p"}]
```



Corrigé de l'exercice 1-1 c)



Corrigé de l'exercice 1-2

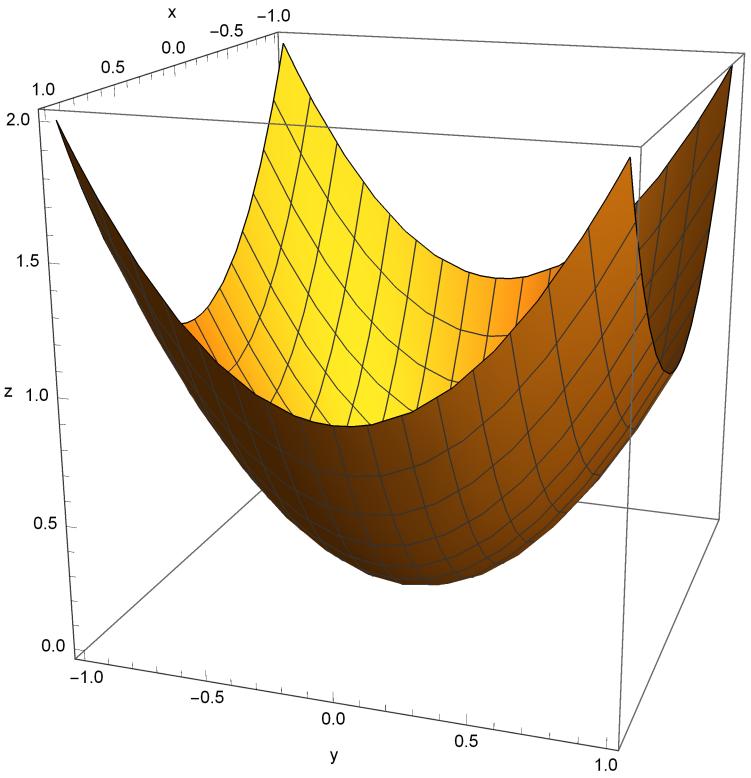
a) Paraboloïde elliptique

$$z = f(x, y) = x^2 + y^2$$

$$D_f = \mathbb{R}^2$$

```
Plot3D[x^2 + y^2, {x, -1, 1}, {y, -1, 1}, ViewPoint → {3, 1, 1},
 AxesLabel → {"x", "y", "z"}, ImageSize → {400, 400}, BoxRatios → Automatic]
```

tracé de surfaces | point de vue spatial | titre d'axe | taille d'image | rapports de b··· | automatique



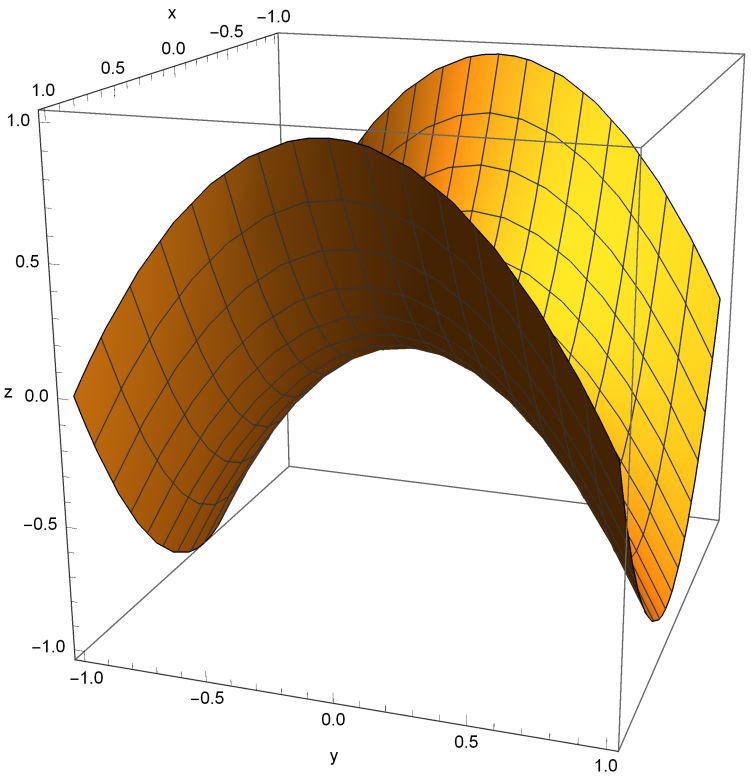
b) Paraboloïde hyperbolique

$$z = f(x, y) = x^2 - y^2$$

$$D_f = \mathbb{R}^2$$

```
Plot3D[x^2 - y^2, {x, -1, 1}, {y, -1, 1}, ViewPoint → {3, 1, 1},
 AxesLabel → {"x", "y", "z"}, ImageSize → {400, 400}, BoxRatios → Automatic]
```

tracé de surfaces | point de vue spatial | titre d'axe | taille d'image | rapports de b··· | automatique



c) Ellipsoïde (cas particulier : la sphère)

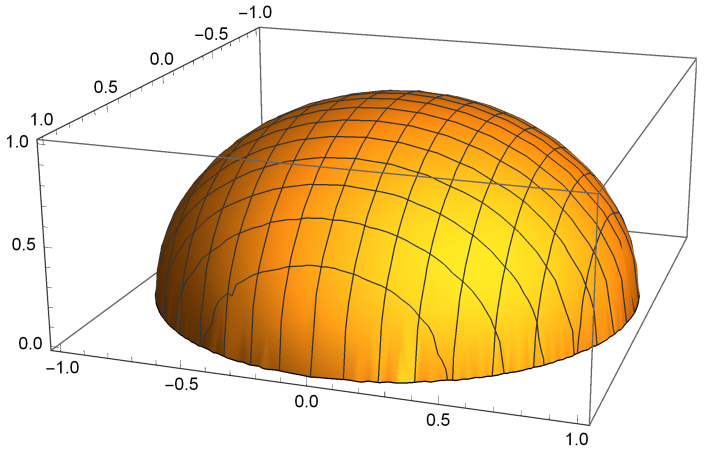
$$z^2 = 1 - x^2 - y^2$$

$$z = f_1(x, y) = \sqrt{1 - x^2 - y^2} \quad \text{ou} \quad z = f_2(x, y) = -\sqrt{1 - x^2 - y^2}$$

$$D_{f_1} = D_{f_2} = \left\{ (x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1 \right\} = \text{disque fermé de centre } (0, 0) \text{ et de rayon } 1$$

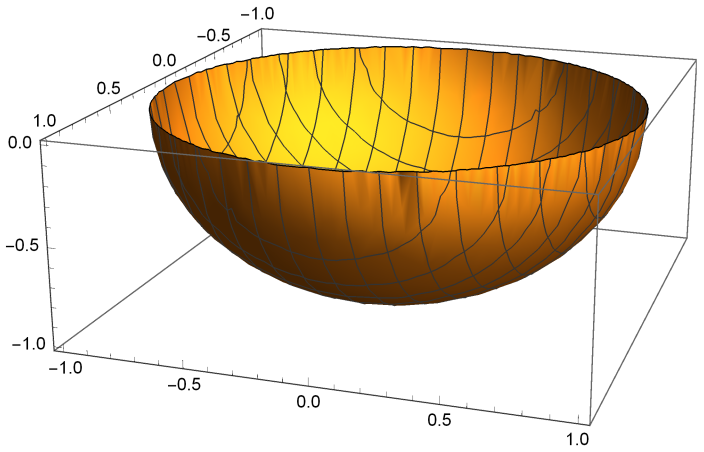
```
f1 = Plot3D[Sqrt[1 - x^2 - y^2], {x, -1, 1}, {y, -1, 1}, ViewPoint → {3, 1, 1}]
```

tracé de surfaces — point de vue spatial



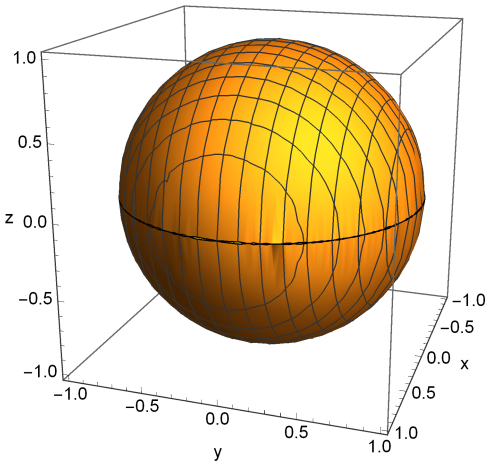

```
f2 = Plot3D[-Sqrt[1 - x^2 - y^2], {x, -1, 1}, {y, -1, 1}, ViewPoint → {3, 1, 1}]
```

tracé de surfaces — point de vue spatial



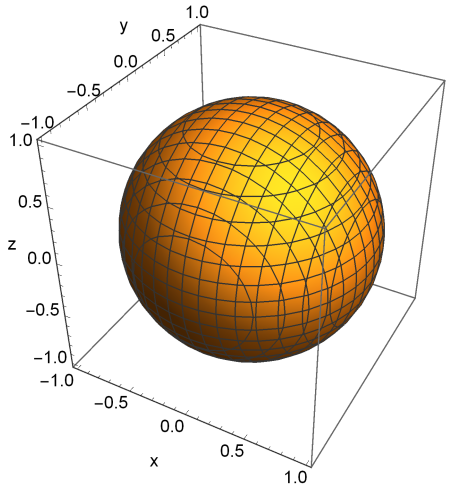
```
Show[f1, f2, AxesLabel → {"x", "y", "z"},
```
montre titre d'axe

```
 ImageSize → {250, 250}, BoxRatios → Automatic, PlotRange → All]
```
taille d'image rapports de b··· automatique zone de tracé tout




```
ContourPlot3D[x^2 + y^2 + z^2 - 1 == 0, {x, -1, 1}, {y, -1, 1},
```
tracé 3D de champ scalaire par ses contours

```
 {z, -1, 1}, AxesLabel → {"x", "y", "z"}, ImageSize → {250, 250}]
```
titre d'axe taille d'image



Corrigé de l'exercice 1-2

d) Hyperboloïde à deux nappes

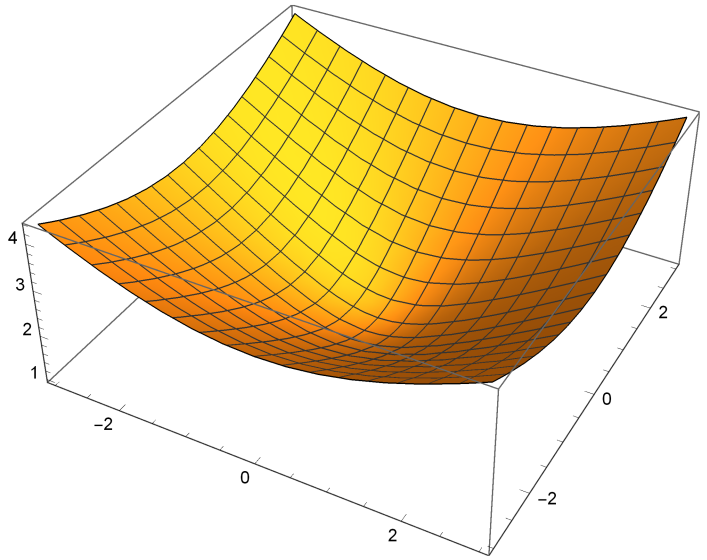
$$z^2 = x^2 + y^2 + 1$$

$$z = f_1(x, y) = \sqrt{x^2 + y^2 + 1} \quad \text{ou} \quad z = f_2(x, y) = -\sqrt{x^2 + y^2 + 1}$$

$$D_{f_1} = D_{f_2} = \mathbb{R}^2$$

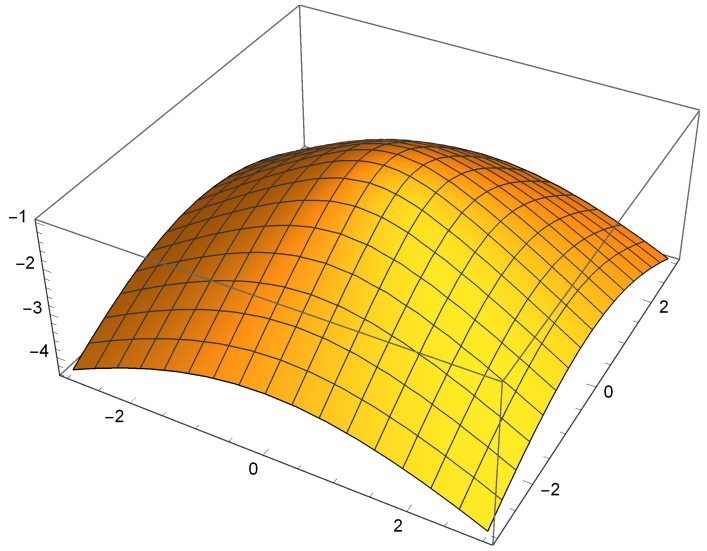
```
f1 = Plot3D[Sqrt[x^2 + y^2 + 1], {x, -3, 3}, {y, -3, 3}]
```

tracé de surfaces



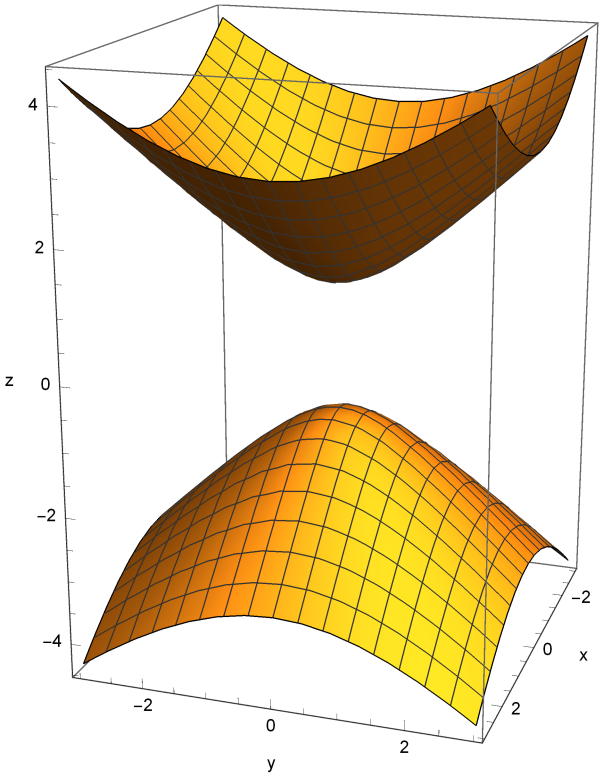

```
f2 = Plot3D[-Sqrt[x^2 + y^2 + 1], {x, -3, 3}, {y, -3, 3}]
```

tracé de surfaces



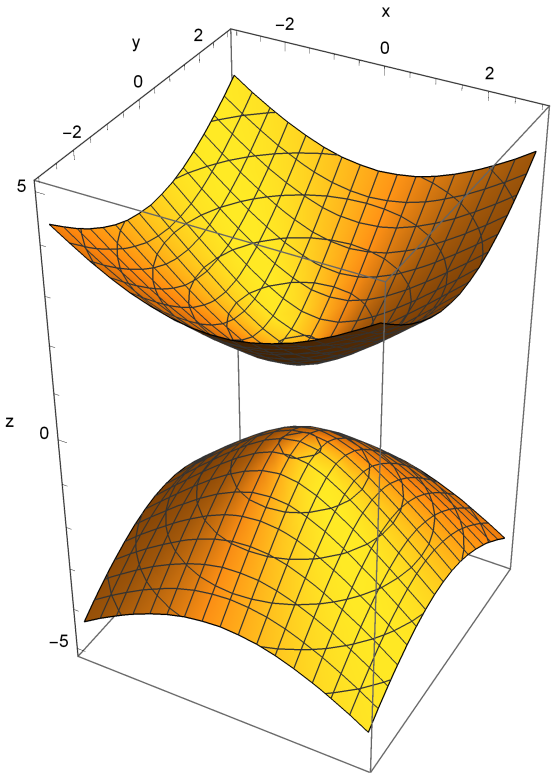
```
Show[f1, f2, ViewPoint → {3, 1, 1}, AxesLabel → {"x", "y", "z"},
 ImageSize → {400, 400}, BoxRatios → Automatic, PlotRange → All]
```

montre | point de vue spatial | titre d'axe | taille d'image | rapports de b··· | automatique | zone de tracé | tout



```
ContourPlot3D[z^2 == x^2 + y^2 + 1, {x, -3, 3}, {y, -3, 3}, {z, -5, 5},
 AxesLabel → {"x", "y", "z"}, BoxRatios → Automatic, ImageSize → {400, 400}]
```

tracé 3D de champ scalaire par ses contours | titre d'axe | rapports de b··· | automatique | taille d'image



e) Cône

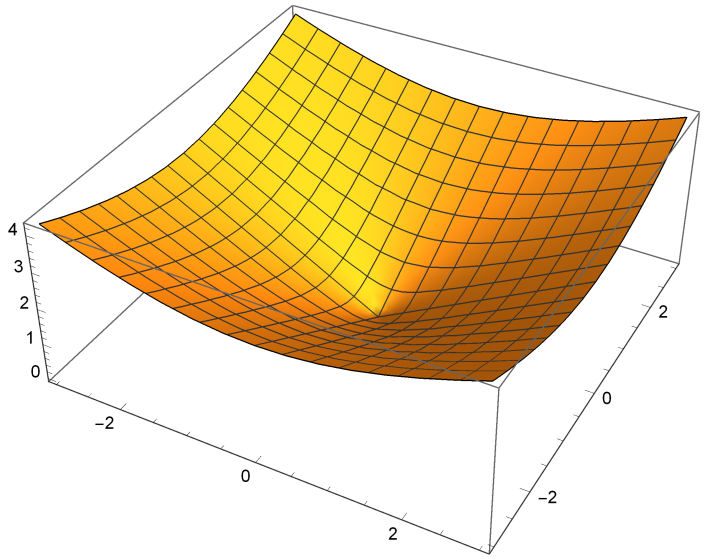
$$z^2 = x^2 + y^2$$

$$z = f_1(x, y) = \sqrt{x^2 + y^2} \quad \text{ou} \quad z = f_2(x, y) = -\sqrt{x^2 + y^2}$$

$$D_{f_1} = D_{f_2} = \mathbb{R}^2$$

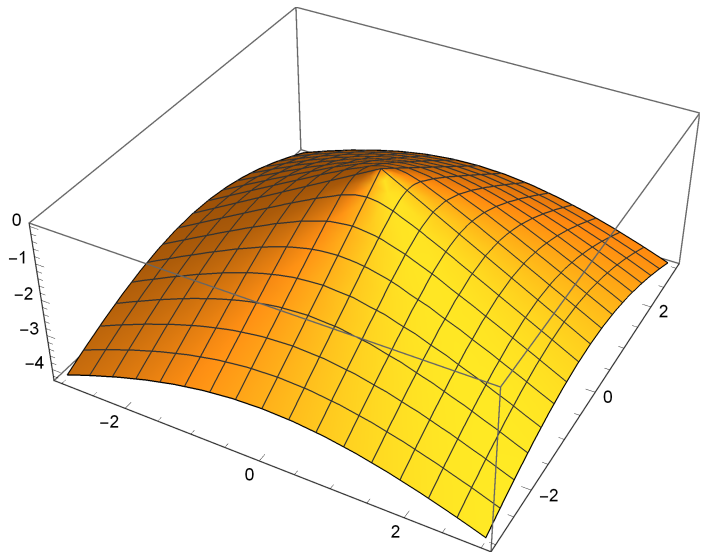
```
f1 = Plot3D[Sqrt[x^2 + y^2], {x, -3, 3}, {y, -3, 3}]
```

tracé de surfaces

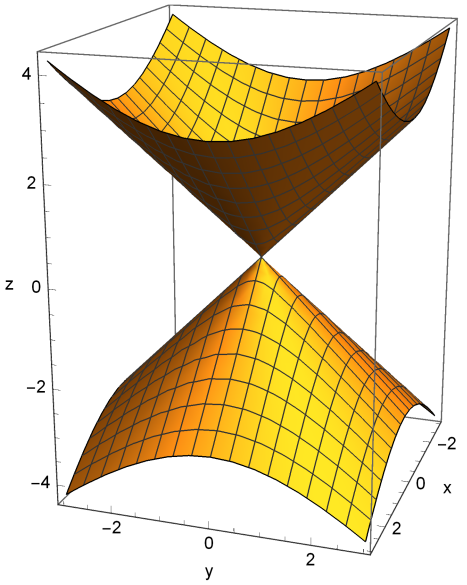



```
f2 = Plot3D[-Sqrt[x^2 + y^2], {x, -3, 3}, {y, -3, 3}]
```

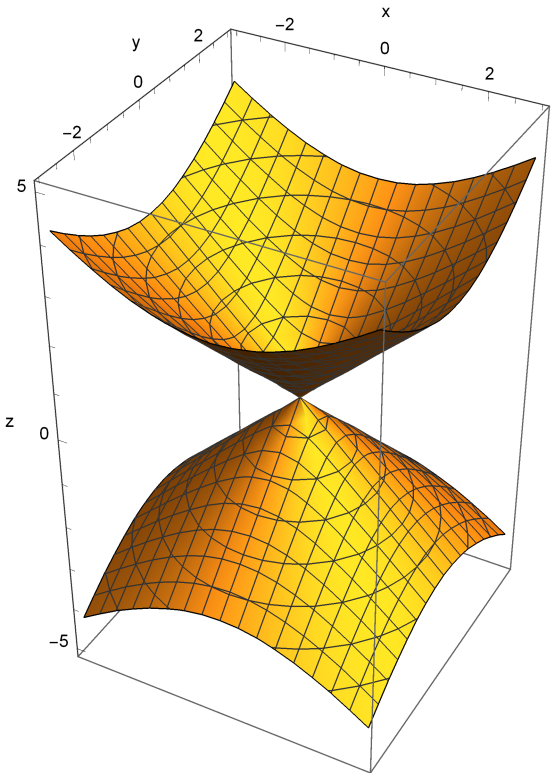
tracé de surfaces



```
Show[f1, f2, ViewPoint → {3, 1, 1}, AxesLabel → {"x", "y", "z"},
```
montre point de vue spatial titre d'axe
```
 ImageSize → {300, 300}, BoxRatios → Automatic, PlotRange → All]
```
taille d'image rapports de b··· automatique zone de tracé tout




```
ContourPlot3D[z^2 == x^2 + y^2, {x, -3, 3}, {y, -3, 3}, {z, -5, 5},
```
tracé 3D de champ scalaire par ses contours
```
 AxesLabel → {"x", "y", "z"}, BoxRatios → Automatic, ImageSize → {400, 400}]
```
titre d'axe rapports de b··· automatique taille d'image



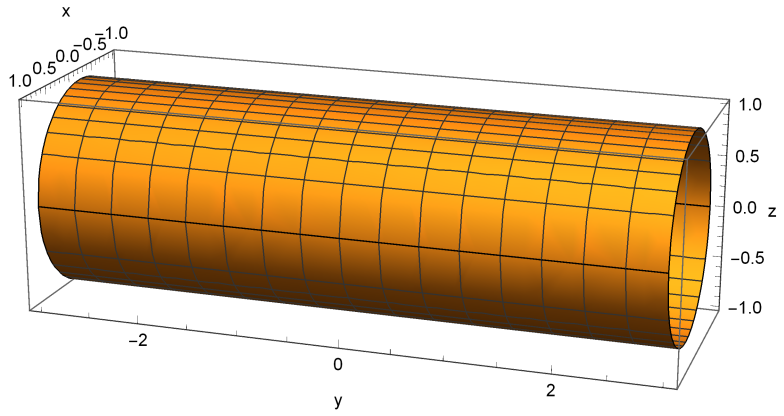
f) Cylindre

$$z^2 = 1 - x^2$$

$$z = f_1(x, y) = \sqrt{1 - x^2} \quad \text{ou} \quad z = f_2(x, y) = -\sqrt{1 - x^2}$$

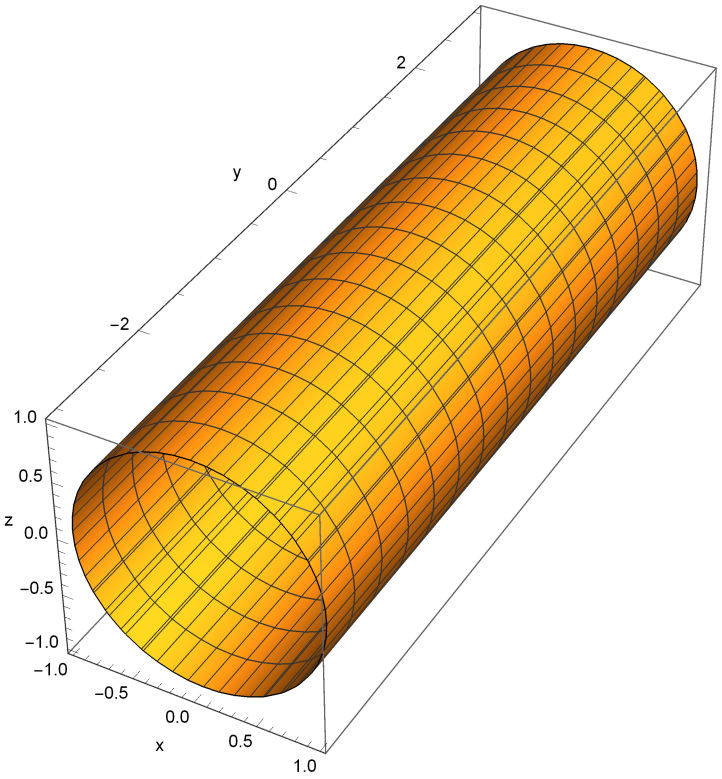
$$D_{f_1} = D_{f_2} = \left\{ (x, y) \in \mathbb{R}^2 \mid -1 \le x \le 1 \text{ et } -\infty < y < \infty \right\}$$

```
Show[Plot3D[Sqrt[1 - x^2], {x, -1, 1}, {y, -3, 3}], Plot3D[-Sqrt[1 - x^2], {x, -1, 1}, {y, -3, 3}],
 ViewPoint → {3, 1, 1}, AxesLabel → {"x", "y", "z"},
 ImageSize → {400, 400}, BoxRatios → Automatic, PlotRange → All]
```



```
ContourPlot3D[z^2 == 1 - x^2, {x, -1, 1}, {y, -3, 3}, {z, -1, 1},
 AxesLabel → {"x", "y", "z"}, BoxRatios → Automatic, ImageSize → {400, 400}]
```

tracé 3D de champ scalaire par ses contours
titre d'axe
rapports de b···
automatique
taille d'image



g) Cylindre parabolique

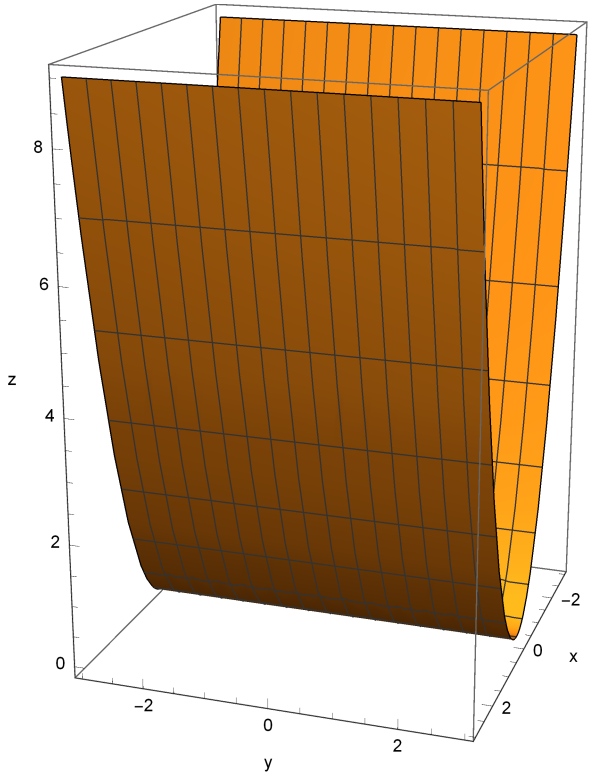
$$z = x^2$$

$$z = f(x, y) = x^2$$

$$D_{f_1} = D_{f_2} = \mathbb{R}^2$$

```
Plot3D[x^2, {x, -3, 3}, {y, -3, 3}, ViewPoint → {3, 1, 1},
 AxesLabel → {"x", "y", "z"}, ImageSize → {400, 400}, BoxRatios → Automatic]
```

tracé de surfaces | point de vue spatial | titre d'axe | taille d'image | rapports de b··· | automatique



Corrigé de l'exercice 1-3 (facultatif)

```
pv = pv = Table[N[{3 Cos[φ], 3 Sin[φ], 2}], {φ, 0, 2 π, π/6}]
```

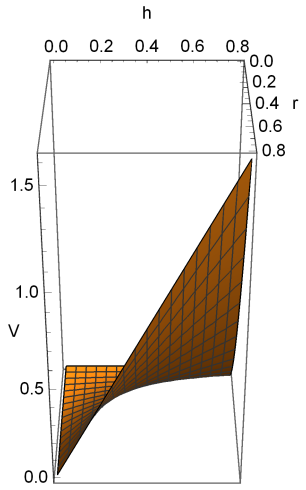
table | val··· | cosinus | sinus

```
{{3., 0., 2.}, {2.59808, 1.5, 2.}, {1.5, 2.59808, 2.}, {0., 3., 2.}, {-1.5, 2.59808, 2.},
 {-2.59808, 1.5, 2.}, {-3., 0., 2.}, {-2.59808, -1.5, 2.}, {-1.5, -2.59808, 2.},
 {0., -3., 2.}, {1.5, -2.59808, 2.}, {2.59808, -1.5, 2.}, {3., 0., 2.}}
```

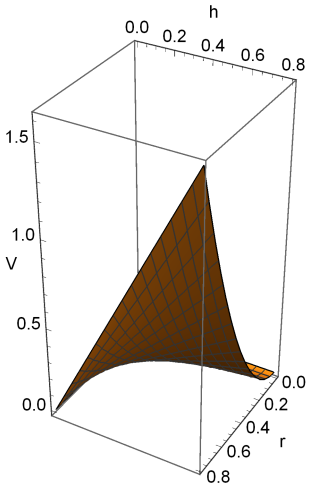
```
Table[Plot3D[π r^2 h, {r, 0, 0.8}, {h, 0, 0.8}, BoxRatios → Automatic, ViewPoint → pv[[j]],
 AxesLabel → {"r", "h", "V"}, ImageSize → {250, 250}], {j, 1, Length[pv]}]
```

table | tracé de surfaces | rapports de b··· | automatique | point de vue spatial | titre d'axe | taille d'image | longueur

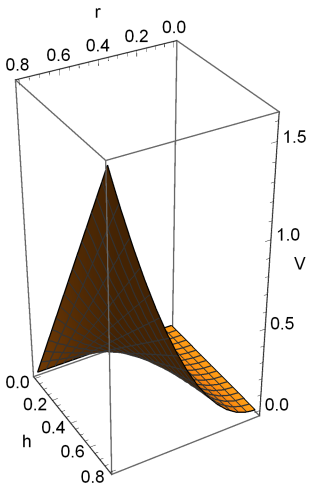
{


h
0.0 0.2 0.4 0.6 0.8
0.0
0.2
0.4 r
0.6
0.8
1.5
1.0
V
0.5
0.0

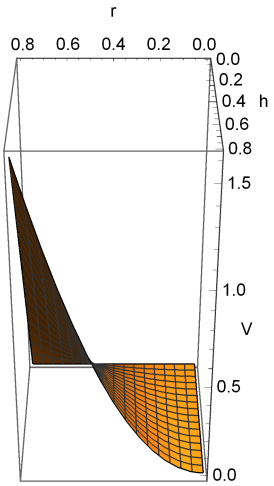
,


h
0.0 0.2 0.4 0.6 0.8
1.5
1.0
V
0.5
0.0
0.0
0.2
0.4
r
0.6
0.8

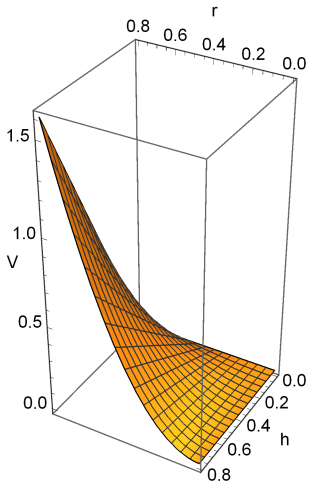
,


r
0.8 0.6 0.4 0.2 0.0
1.5
1.0
V
0.5
0.0
0.0
0.2
h
0.4
0.6
0.8

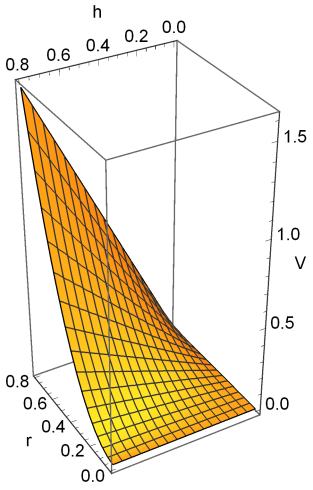
,


r
0.8 0.6 0.4 0.2 0.0
0.0
0.2
0.4 h
0.6
0.8
1.5
1.0
V
0.5
0.0

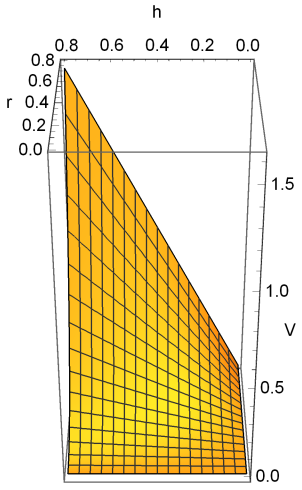
,


r
0.8 0.6 0.4 0.2 0.0
1.5
1.0
V
0.5
0.0
0.0
0.2
0.4
h
0.6
0.8

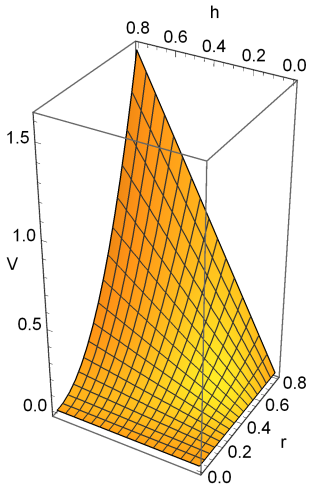
,


h
0.0 0.2 0.4 0.6 0.8
r
0.0 0.2 0.4 0.6 0.8
V
0.0 0.5 1.0 1.5

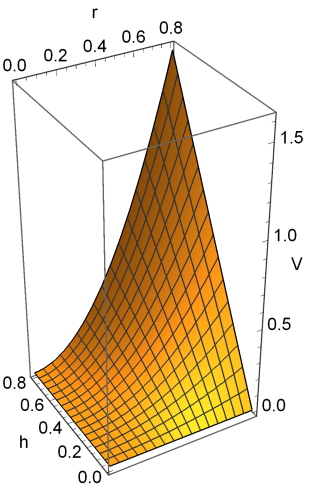
,


h
0.8 0.6 0.4 0.2 0.0
r
0.0 0.2 0.4 0.6 0.8
V
0.0 0.5 1.0 1.5

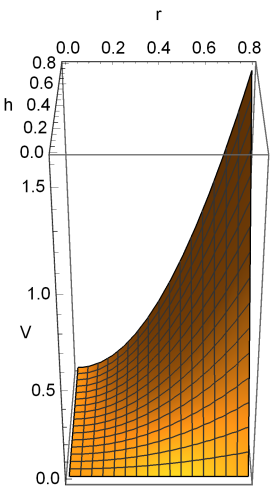
,


h
0.8 0.6 0.4 0.2 0.0
V
0.0 0.5 1.0 1.5
r
0.0 0.2 0.4 0.6 0.8

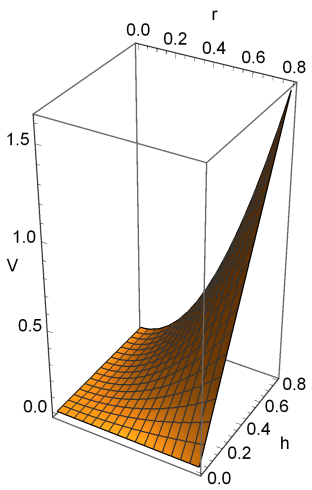
,


r
0.0 0.2 0.4 0.6 0.8
V
0.0 0.5 1.0 1.5
h
0.0 0.2 0.4 0.6 0.8

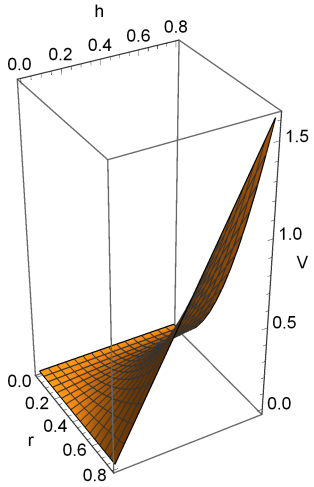
,


r
0.0 0.2 0.4 0.6 0.8
h
0.0 0.2 0.4 0.6 0.8
V
0.0 0.5 1.0 1.5

,


r
0.0 0.2 0.4 0.6 0.8
V
0.0 0.5 1.0 1.5
h
0.0 0.2 0.4 0.6 0.8

,



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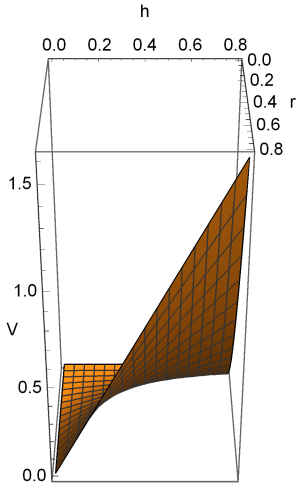

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 AxesLabel → {"x", "y", "z=f(x,y)"}, ImageSize → {250, 250}], {j, 1, Length[pv]}]
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table | tracé de surfaces | rapports de b… | automatique | point de vue spatial | titre d'axe | taille d'image | longueur

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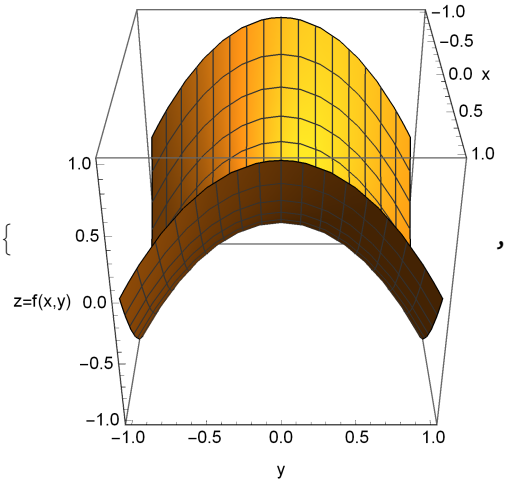

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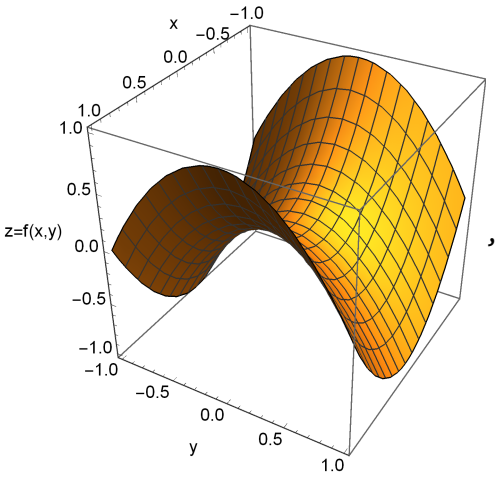

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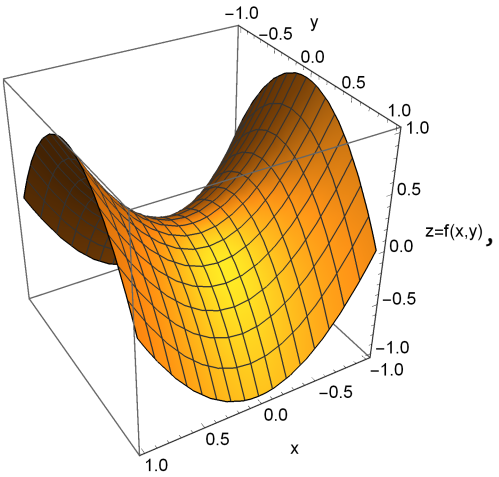

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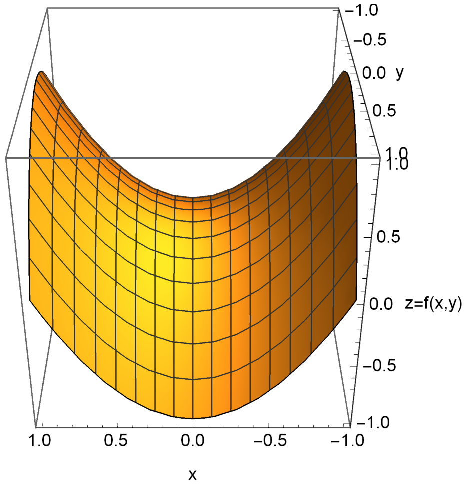
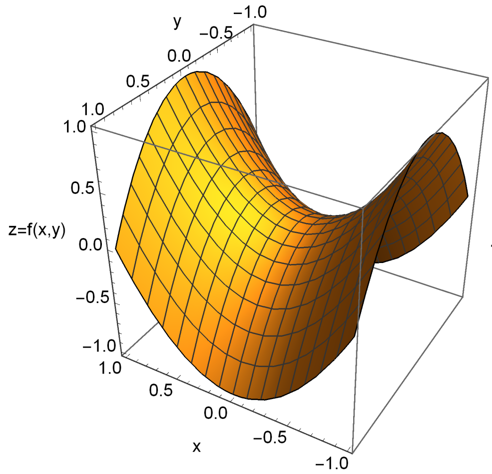


x
y
z=f(x,y)

,


x
y
z=f(x,y)

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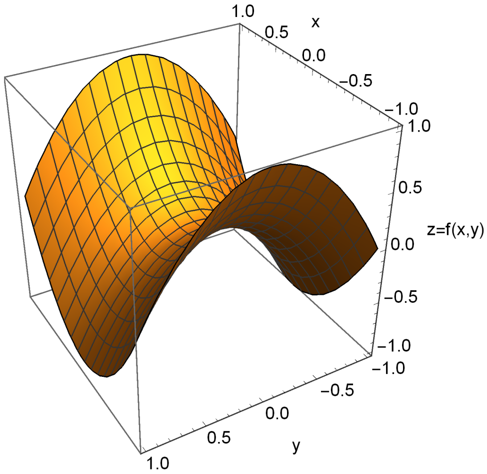

x
y
z=f(x,y)

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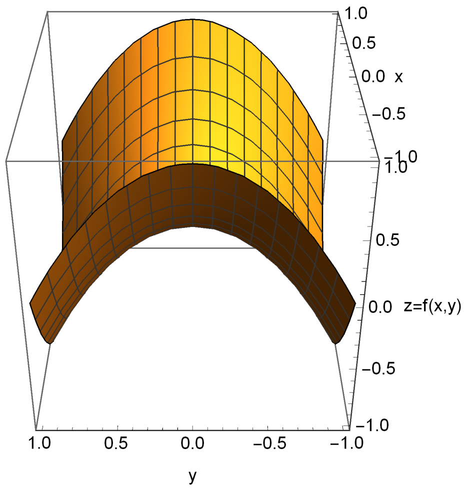
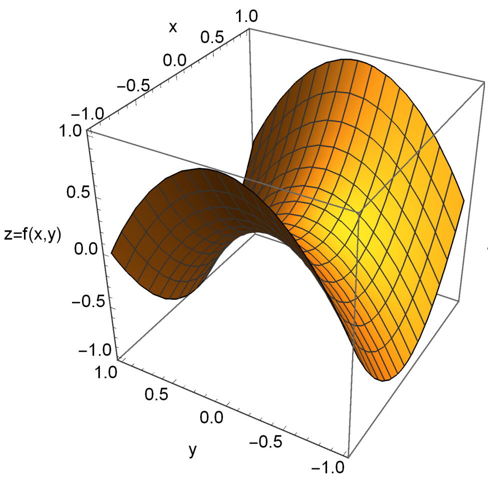


x
y
z=f(x,y)

,


x
y
z=f(x,y)

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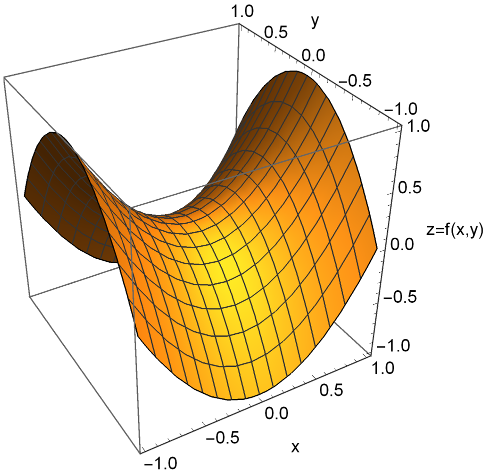
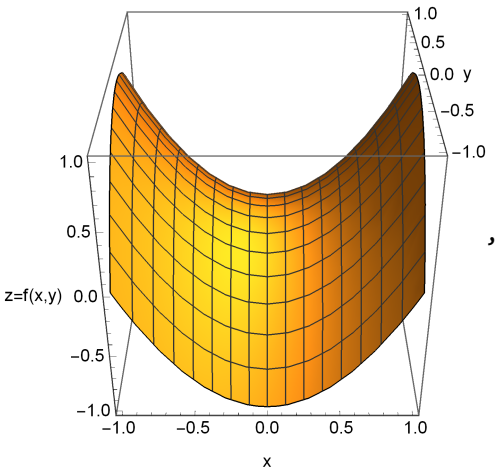


x
y
z=f(x,y)

,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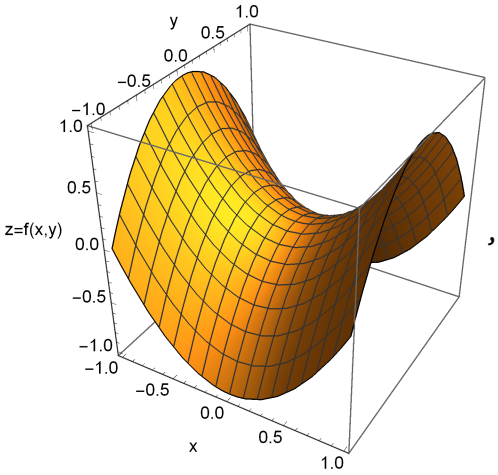
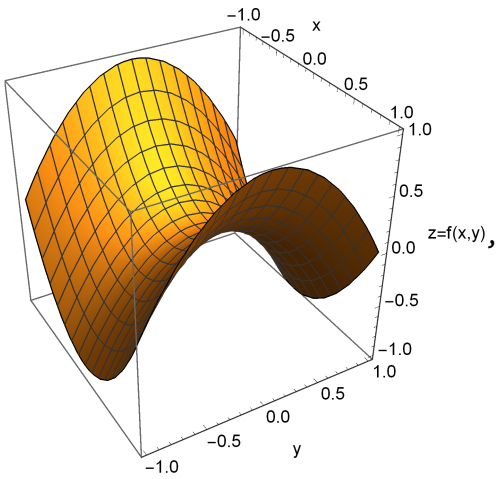
x
y
1.0
0.5
0.0
−0.5
−1.0

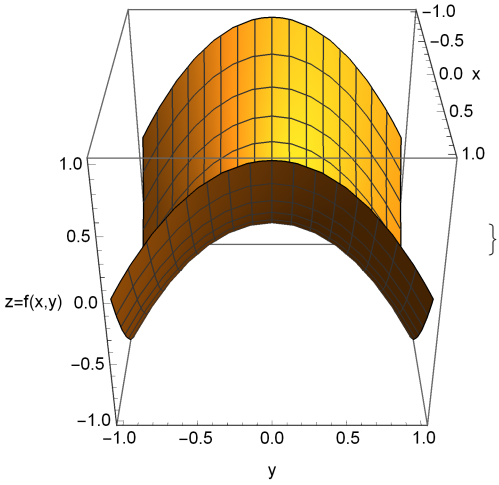
,


z=f(x,y)
x
y
1.0
0.5
0.0
−0.5
−1.0

,


z=f(x,y)
x
y
1.0
0.5
0.0
−0.5
−1.0

,


z=f(x,y)
x
y
1.0
0.5
0.0
−0.5
−1.0

}