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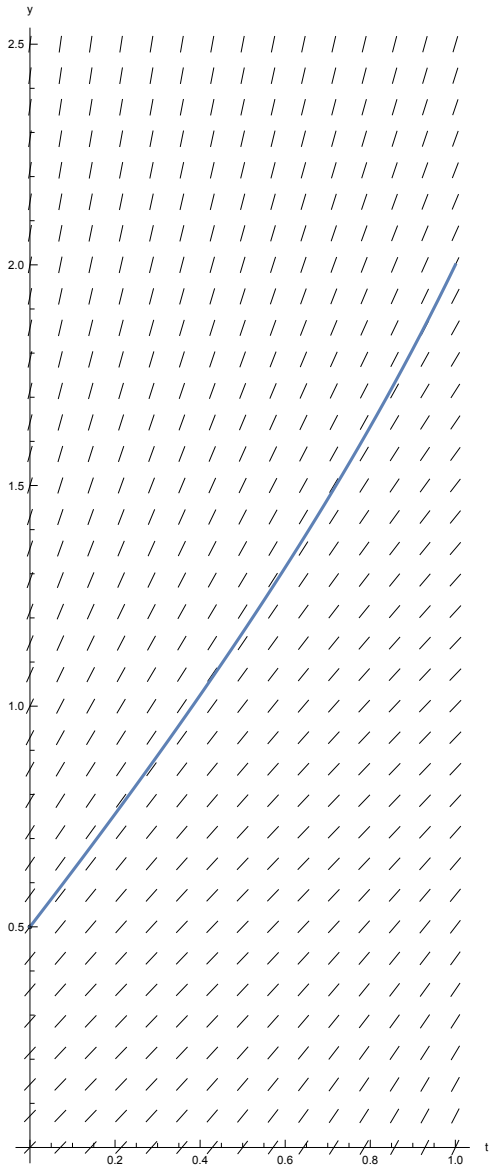
Edition 2017

Thème : § 1-2 Résolution numérique d'une équation différentielle ordinaire du premier ordre
Lien vers les énoncés des travaux dirigés:
www.deleze.name/marcel/sec2/applmaths/csud/eq-differentielles/1-2_EQ-DIFFERENTIELLES.pdf

Corrigé de 1.2- TD 1

a) Résolution graphique



b) Méthode d'Euler sans ordinateur

$$n = 10$$

$$h = \frac{1 - 0}{10} = 0.1$$

$$t_0 = 0$$

$$y_0 = 0.5$$

$$t_1 = t_0 + h = 0.1$$

$$y_1 = y_0 + h\, f(t_0, y_0) = y_0 + h\left(1 + (y_0 - t_0)^2\right) = 0.5 + 0.1\left(1 + (0.5 - 0)^2\right) \simeq 0.625$$

$$t_2 = t_1 + h = 0.2$$

$$y_2 =$$

$$y_1 + h\, f(t_1, y_1) = y_1 + h\left(1 + (y_1 - t_1)^2\right) = 0.625 + 0.1\left(1 + (0.625 - 0.1)^2\right) \simeq 0.7525625$$

$$\ldots$$

t	*y*
0.	0.5
0.1	0.625
0.2	0.752563
0.3	0.883095
0.4	1.0171
0.5	1.15518
0.6	1.2981
0.7	1.44684
0.8	1.60261
0.9	1.76703
1.	1.9422

c) Méthode d'Euler avec ordinateur

```
f[t_, y_] := 1 + (y - t)^2
```

```
t0 = 0.; y0 = 0.5; tmax = 1; n = 100; h = (tmax - t0)/n;
```

Partez d'une condition initiale exprimée en virgule flottante (remarquez la présence du point décimal) :

```
euler[{t_, y_}] := {t + h, y + h f[t, y]}
```

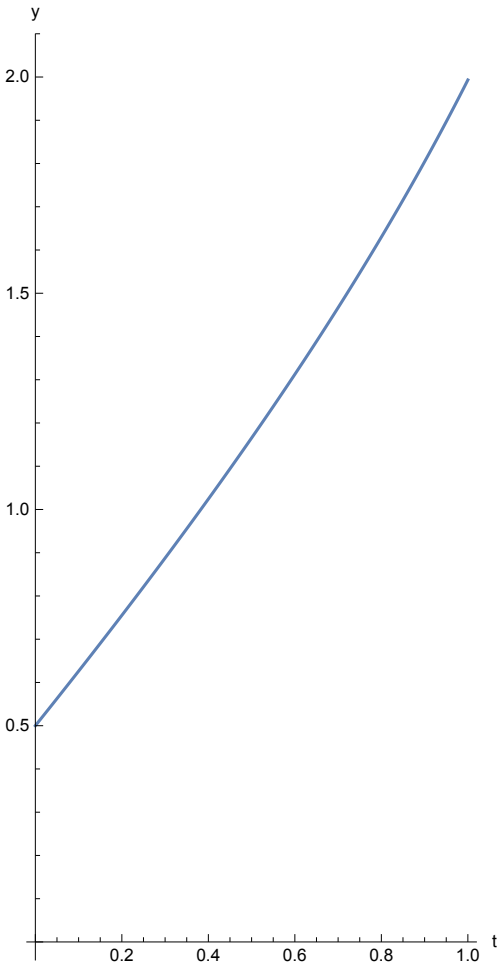
```
solC = NestList[euler, {t0, y0}, n]
```

liste d'imbrication

```
{{0., 0.5}, {0.01, 0.5125}, {0.02, 0.525025}, {0.03, 0.537576}, {0.04, 0.550152},
 {0.05, 0.562754}, {0.06, 0.575384}, {0.07, 0.58804}, {0.08, 0.600723},
 {0.09, 0.613435}, {0.1, 0.626175}, {0.11, 0.638943}, {0.12, 0.651741},
 {0.13, 0.664569}, {0.14, 0.677426}, {0.15, 0.690315}, {0.16, 0.703234},
 {0.17, 0.716185}, {0.18, 0.729168}, {0.19, 0.742184}, {0.2, 0.755233},
 {0.21, 0.768316}, {0.22, 0.781433}, {0.23, 0.794585}, {0.24, 0.807773},
 {0.25, 0.820996}, {0.26, 0.834257}, {0.27, 0.847555}, {0.28, 0.86089},
 {0.29, 0.874265}, {0.3, 0.887678}, {0.31, 0.901132}, {0.32, 0.914626},
 {0.33, 0.928162}, {0.34, 0.94174}, {0.35, 0.955361}, {0.36, 0.969026},
 {0.37, 0.982735}, {0.38, 0.996489}, {0.39, 1.01029}, {0.4, 1.02414},
 {0.41, 1.03803}, {0.42, 1.05198}, {0.43, 1.06597}, {0.44, 1.08002}, {0.45, 1.09411},
 {0.46, 1.10826}, {0.47, 1.12246}, {0.48, 1.13672}, {0.49, 1.15103}, {0.5, 1.1654},
 {0.51, 1.17983}, {0.52, 1.19432}, {0.53, 1.20886}, {0.54, 1.22347}, {0.55, 1.23814},
 {0.56, 1.25288}, {0.57, 1.26768}, {0.58, 1.28255}, {0.59, 1.29748}, {0.6, 1.31249},
 {0.61, 1.32757}, {0.62, 1.34271}, {0.63, 1.35794}, {0.64, 1.37324}, {0.65, 1.38861},
 {0.66, 1.40407}, {0.67, 1.4196}, {0.68, 1.43522}, {0.69, 1.45093}, {0.7, 1.46672},
 {0.71, 1.4826}, {0.72, 1.49856}, {0.73, 1.51463}, {0.74, 1.53078}, {0.75, 1.54704},
 {0.76, 1.56339}, {0.77, 1.57984}, {0.78, 1.5964}, {0.79, 1.61307}, {0.8, 1.62984},
 {0.81, 1.64673}, {0.82, 1.66373}, {0.83, 1.68085}, {0.84, 1.69809}, {0.85, 1.71545},
 {0.86, 1.73294}, {0.87, 1.75056}, {0.88, 1.76831}, {0.89, 1.78621}, {0.9, 1.80424},
 {0.91, 1.82241}, {0.92, 1.84074}, {0.93, 1.85922}, {0.94, 1.87785}, {0.95, 1.89665},
 {0.96, 1.91561}, {0.97, 1.93474}, {0.98, 1.95405}, {0.99, 1.97353}, {1., 1.99321}}
```

```
ListLinePlot[solC, AspectRatio → Automatic,
 AxesOrigin → {0, 0}, ImageSize → {300, 500}, AxesLabel → {"t", "y"}]
```

tracé de liste de ligne rapport d'aspect automatique
origine des axes taille d'image titre d'axe



d) Résolution numérique avec *Mathematica*

```
f[t_, y_] := 1 + (y - t)^2
```

```
t0 = 0; tmax = 1;  y0 = 0.5;
```

```
soln = NDSolve[{y'[t] == f[t, y[t]], y[t0] == y0}, y, {t, t0, tmax}]
```

résolveur numérique d'équations différentielles

```
{{y → InterpolatingFunction[ Domain: {{0., 1.}}  Output: scalar ]}}
```

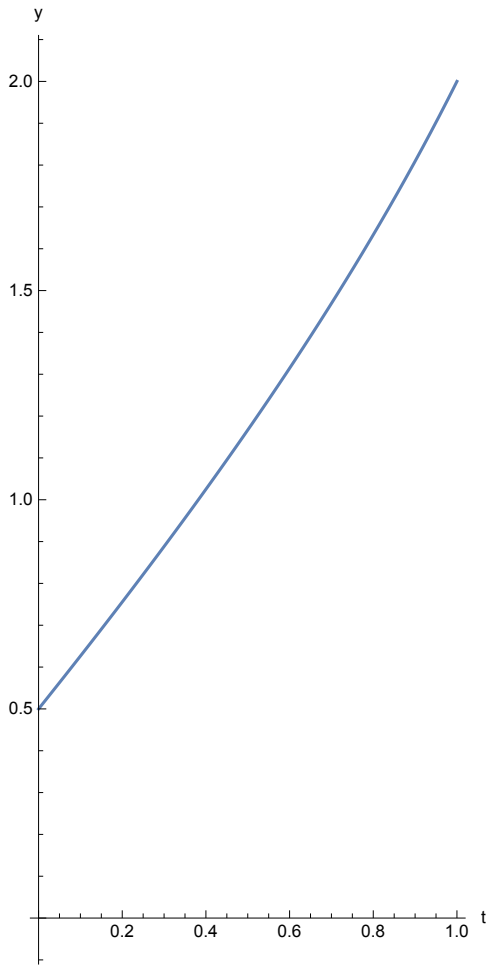
```
sn = y /. soln[[1]]
```

```
InterpolatingFunction[ Domain: {{0., 1.}}  Output: scalar ]
```

```
solD = sn[Range[0, 1, 1/100]]
          plage
```

```
{0.5, 0.512513, 0.525051, 0.537614, 0.550204, 0.562821, 0.575464, 0.588135,
 0.600833, 0.61356, 0.626316, 0.639101, 0.651915, 0.664759, 0.677634, 0.690541,
 0.703478, 0.716448, 0.729451, 0.742486, 0.755556, 0.768659, 0.781798, 0.794972,
 0.808182, 0.821429, 0.834713, 0.848035, 0.861395, 0.874795, 0.888235, 0.901716,
 0.915238, 0.928802, 0.94241, 0.956061, 0.969756, 0.983497, 0.997284, 1.01112,
 1.025, 1.03893, 1.05291, 1.06694, 1.08103, 1.09516, 1.10935, 1.12359, 1.13789,
 1.15225, 1.16667, 1.18114, 1.19568, 1.21027, 1.22493, 1.23966, 1.25444,
 1.2693, 1.28423, 1.29922, 1.31429, 1.32942, 1.34464, 1.35993, 1.37529,
 1.39074, 1.40627, 1.42188, 1.43758, 1.45336, 1.46923, 1.48519, 1.50125, 1.5174,
 1.53365, 1.55, 1.56645, 1.58301, 1.59967, 1.61645, 1.63333, 1.65034, 1.66746,
 1.6847, 1.70207, 1.71957, 1.73719, 1.75496, 1.77286, 1.7909, 1.80909, 1.82743,
 1.84593, 1.86458, 1.8834, 1.90238, 1.92154, 1.94087, 1.96039, 1.9801, 2.}
```

```
Plot[sn[t], {t, t0, tmax}, AspectRatio → Automatic,
 tracé de courbes          rapport d'aspect  automatique
 AxesOrigin → {0, 0}, ImageSize → {300, 500}, AxesLabel → {"t", "y"}]
  origine des axes          taille d'image          titre d'axe
```



e) Vérification

D'une part,

$$y'(t) = \left(t + \frac{1}{2-t}\right)' = 1 + (-1)(2-t)^{-2}(-1) = 1 + \frac{1}{(2-t)^2}$$

D'autre part,

$$1 + (y(t) - t)^2 = 1 + \left(\left(t + \frac{1}{2-t}\right) - t\right)^2 = 1 + \left(\frac{1}{2-t}\right)^2 = 1 + \frac{1}{(2-t)^2}$$

Pour la condition initiale,

$$y(0) = 0 + \frac{1}{2-0} = \frac{1}{2}$$

En conclusion, l'équation différentielle avec condition initiale est satisfaite.

f) Comparaison

Différence entre les méthodes b) et c) : les écarts sont inférieurs à 0.052

```
Transpose[solB][[2]] - Transpose[solC][[2]][[Range[1, 101, 10]]]
transposée              transposée            plage
```

```
{0., -0.00117484, -0.00267069, -0.00458323, -0.00704233,
 -0.0102269, -0.0143876, -0.0198815, -0.0272291, -0.0372063, -0.0510027}
```

Différence entre les méthodes b) et d) : les écarts sont inférieurs à 0.06

```
Transpose[solB][[2]] - sn[Range[0, 1, 1/10]]
transposée              plage
```

```
{0., -0.00131579, -0.00299306, -0.00514026, -0.00790498,
 -0.011491, -0.0161845, -0.0223951, -0.0307214, -0.0420604, -0.0577954}
```

Différence entre les méthodes c) et d) : les écarts, en valeurs absolues, sont inférieurs à 0.007

```
Transpose[solC][[2]] - solD
transposée
```

```
{0., -0.000012575, -0.0000254602, -0.0000386487, -0.0000521868, -0.0000660697,
 -0.0000803048, -0.0000949007, -0.000109868, -0.000125213, -0.000140952,
 -0.000157094, -0.000173644, -0.00019062, -0.000208034, -0.000225893, -0.000244215,
 -0.000263012, -0.000282292, -0.000302072, -0.00032237, -0.000343193, -0.00036456,
 -0.00038649, -0.000408987, -0.000432071, -0.000455773, -0.000480115, -0.0005051,
 -0.000530729, -0.000557036, -0.000584059, -0.00061182, -0.000640324, -0.000669576,
 -0.00069962, -0.000730497, -0.000762227, -0.000794813, -0.000828269, -0.000862652,
 -0.000898005, -0.000934348, -0.000971682, -0.00101004, -0.00104948, -0.00109005,
 -0.00113178, -0.00117466, -0.00121874, -0.0012641, -0.00131079, -0.00135883,
 -0.0014082, -0.001459, -0.0015113, -0.00156518, -0.00162067, -0.00167776,
 -0.00173648, -0.00179696, -0.00185931, -0.00192359, -0.0019898, -0.00205795,
 -0.00212816, -0.00220056, -0.00227527, -0.00235233, -0.00243175, -0.00251356,
 -0.00259797, -0.00268514, -0.00277516, -0.00286807, -0.00296385, -0.0030627,
 -0.00316484, -0.00327044, -0.00337959, -0.00349228, -0.0036086, -0.00372886,
 -0.00385332, -0.00398214, -0.00411538, -0.00425303, -0.0043954, -0.00454286,
 -0.0046957, -0.00485405, -0.00501794, -0.00518757, -0.00536343, -0.00554586,
 -0.00573509, -0.00593117, -0.00613448, -0.00634561, -0.00656494, -0.00679267}
```

```
{Max[%], Min[%]}
 maximum  minimum
```

```
{0., -0.00679267}
```

g) Erreur d'approximation

```
y[t_] := t + 1/(2 - t)
```

Erreur sur la méthode b) : les écarts sont inférieurs à 0.06

```
Transpose[solB][[2]] - y[Range[0, 1, 1/10]]
```
transposée plage

```
{0., -0.00131579, -0.00299306, -0.00514026, -0.00790499,
 -0.011491, -0.0161846, -0.0223951, -0.0307213, -0.0420603, -0.0577952}
```

Erreur sur la méthode c) : les écarts sont inférieurs à 0.07

```
Transpose[solC][[2]] - y[Range[0, 1, 1/100]]
```
transposée plage

```
{0., -0.0000125628, -0.0000254426, -0.0000386476, -0.0000521864, -0.0000660681,
 -0.0000803016, -0.0000948964, -0.000109862, -0.000125209, -0.000140947,
 -0.000157087, -0.00017364, -0.000190617, -0.00020803, -0.000225891, -0.000244212,
 -0.000263006, -0.000282287, -0.000302067, -0.000322362, -0.000343185, -0.000364552,
 -0.000386478, -0.00040898, -0.000432073, -0.000455775, -0.000480104, -0.000505078,
 -0.000530716, -0.000557037, -0.000584062, -0.000611811, -0.000640307, -0.000669572,
 -0.000699628, -0.0007305, -0.000762214, -0.000794793, -0.000828266, -0.00086266,
 -0.000898004, -0.000934326, -0.000971659, -0.00101003, -0.00104948, -0.00109004,
 -0.00113175, -0.00117463, -0.00121874, -0.0012641, -0.00131077, -0.00135878,
 -0.00140818, -0.00145902, -0.00151134, -0.00156519, -0.00162063, -0.00167771,
 -0.00173648, -0.00179701, -0.00185935, -0.00192358, -0.00198975, -0.00205793,
 -0.0021282, -0.00220063, -0.0022753, -0.00235229, -0.00243169, -0.00251358,
 -0.00259806, -0.00268522, -0.00277516, -0.00286799, -0.00296382, -0.00306277,
 -0.00316495, -0.00327049, -0.00337953, -0.00349219, -0.00360863, -0.00372899,
 -0.00385344, -0.00398214, -0.00411526, -0.00425299, -0.00439551, -0.00454304,
 -0.00469578, -0.00485395, -0.00501779, -0.00518754, -0.00536346, -0.00554583,
 -0.00573492, -0.00593104, -0.00613451, -0.00634566, -0.00656485, -0.00679244}
```

Erreur sur la méthode d) : les écarts sont inférieurs à 0.00003

```
sn[Range[0, 1, 1/100]] - y[Range[0, 1, 1/100]]
 plage                     plage
```

$$\{0.,\ 1.22157\times10^{-8},\ 1.76217\times10^{-8},\ 1.14335\times10^{-9},\ 4.02329\times10^{-10},\ 1.61103\times10^{-9},$$
$$3.25965\times10^{-9},\ 4.29742\times10^{-9},\ 5.70891\times10^{-9},\ 4.36125\times10^{-9},\ 5.38685\times10^{-9},$$
$$7.47785\times10^{-9},\ 4.69496\times10^{-9},\ 3.0516\times10^{-9},\ 4.11376\times10^{-9},\ 2.71849\times10^{-9},$$
$$3.36957\times10^{-9},\ 6.26862\times10^{-9},\ 4.98959\times10^{-9},\ 5.39096\times10^{-9},\ 8.66944\times10^{-9},$$
$$7.59334\times10^{-9},\ 7.68536\times10^{-9},\ 1.13057\times10^{-8},\ 7.27584\times10^{-9},\ -2.52766\times10^{-9},$$
$$-2.19612\times10^{-9},\ 1.03778\times10^{-8},\ 2.22516\times10^{-8},\ 1.33538\times10^{-8},\ -1.12539\times10^{-9},$$
$$-3.1994\times10^{-9},\ 8.24213\times10^{-9},\ 1.68886\times10^{-8},\ 4.22792\times10^{-9},\ -8.00453\times10^{-9},$$
$$-3.76601\times10^{-9},\ 1.33585\times10^{-8},\ 1.97907\times10^{-8},\ 3.05244\times10^{-9},\ -7.79916\times10^{-9},$$
$$1.44128\times10^{-9},\ 2.20865\times10^{-8},\ 2.34565\times10^{-8},\ 2.8187\times10^{-9},\ -5.82228\times10^{-9},$$
$$9.11132\times10^{-9},\ 3.23659\times10^{-8},\ 2.65889\times10^{-8},\ 2.78099\times10^{-9},\ -2.03233\times10^{-9},$$
$$1.98148\times10^{-8},\ 4.43762\times10^{-8},\ 2.28019\times10^{-8},\ -2.15342\times10^{-8},\ -3.74484\times10^{-8},$$
$$-9.46209\times10^{-9},\ 4.04515\times10^{-8},\ 5.21436\times10^{-8},\ 8.18183\times10^{-11},\ -4.50232\times10^{-8},$$
$$-4.11582\times10^{-8},\ 9.05432\times10^{-9},\ 5.67274\times10^{-8},\ 2.46681\times10^{-8},\ -4.30582\times10^{-8},$$
$$-6.94856\times10^{-8},\ -3.0703\times10^{-8},\ 4.16345\times10^{-8},\ 5.89616\times10^{-8},\ -1.75023\times10^{-8},$$
$$-8.24675\times10^{-8},\ -7.33178\times10^{-8},\ 5.35959\times10^{-9},\ 7.91067\times10^{-8},\ 3.0814\times10^{-8},$$
$$-7.18732\times10^{-8},\ -1.11158\times10^{-7},\ -5.03835\times10^{-8},\ 6.18317\times10^{-8},\ 8.9094\times10^{-8},$$
$$-2.97679\times10^{-8},\ -1.30953\times10^{-7},\ -1.1633\times10^{-7},\ 7.28864\times10^{-9},\ 1.23584\times10^{-7},$$
$$4.75721\times10^{-8},\ -1.15388\times10^{-7},\ -1.77364\times10^{-7},\ -7.94393\times10^{-8},\ 1.01068\times10^{-7},$$
$$1.5172\times10^{-7},\ 3.08233\times10^{-8},\ -3.73238\times10^{-8},\ 3.55664\times10^{-8},\ 1.66591\times10^{-7},$$
$$1.24686\times10^{-7},\ -2.60918\times10^{-8},\ -4.94788\times10^{-8},\ 9.24772\times10^{-8},\ 2.20248\times10^{-7}\}$$

Corrigé de 1.2- TD 2 a)

La méthode de Heun, d'après *Formulaires et tables*

$$x_{n+1} = x_n + h;$$

$$y_{n+1} = y_n + \frac{p_n + q_n}{2} h \qquad \text{où} \qquad \begin{cases} p_n = f(x_n, y_n) \\ q_n = f(x_n + h, y_n + p_n h) \end{cases}$$

Avec *Mathematica*

```
Clear[f, t, y]; f[t_, y_] := 4 t / (y + 1)
```
efface

```
t0 = 0.; y0 = 1.;   tmax = 2; n = 100; h = (tmax - t0) / n;

heun[{t_, y_}] := Module[{p, q}, p = f[t, y]; q = f[t + h, y + p h];
  {t + h, y + (p + q) / 2 h}]
```
module

```
solH = NestList[heun, {t0, y0}, n]
```
liste d'imbrication

```
{{0., 1.}, {0.02, 1.0004}, {0.04, 1.0016}, {0.06, 1.0036}, {0.08, 1.00639},
 {0.1, 1.00998}, {0.12, 1.01435}, {0.14, 1.01951}, {0.16, 1.02544},
 {0.18, 1.03214}, {0.2, 1.03961}, {0.22, 1.04783}, {0.24, 1.05679},
 {0.26, 1.0665}, {0.28, 1.07692}, {0.3, 1.08806}, {0.32, 1.09991}, {0.34, 1.11244},
 {0.36, 1.12565}, {0.38, 1.13953}, {0.4, 1.15407}, {0.42, 1.16924},
 {0.44, 1.18504}, {0.46, 1.20145}, {0.48, 1.21847}, {0.5, 1.23607}, {0.52, 1.25424},
 {0.54, 1.27297}, {0.56, 1.29225}, {0.58, 1.31206}, {0.6, 1.33238}, {0.62, 1.35321},
 {0.64, 1.37453}, {0.66, 1.39633}, {0.68, 1.4186}, {0.7, 1.44131}, {0.72, 1.46447},
 {0.74, 1.48805}, {0.76, 1.51205}, {0.78, 1.53646}, {0.8, 1.56125}, {0.82, 1.58643},
 {0.84, 1.61197}, {0.86, 1.63788}, {0.88, 1.66413}, {0.9, 1.69073}, {0.92, 1.71765},
 {0.94, 1.74489}, {0.96, 1.77244}, {0.98, 1.80029}, {1., 1.82843}, {1.02, 1.85685},
 {1.04, 1.88555}, {1.06, 1.91452}, {1.08, 1.94374}, {1.1, 1.97321}, {1.12, 2.00293},
 {1.14, 2.03289}, {1.16, 2.06307}, {1.18, 2.09348}, {1.2, 2.1241}, {1.22, 2.15493},
 {1.24, 2.18597}, {1.26, 2.2172}, {1.28, 2.24863}, {1.3, 2.28024}, {1.32, 2.31204},
 {1.34, 2.34401}, {1.36, 2.37615}, {1.38, 2.40846}, {1.4, 2.44093}, {1.42, 2.47356},
 {1.44, 2.50634}, {1.46, 2.53927}, {1.48, 2.57234}, {1.5, 2.60555}, {1.52, 2.6389},
 {1.54, 2.67238}, {1.56, 2.706}, {1.58, 2.73973}, {1.6, 2.77359}, {1.62, 2.80757},
 {1.64, 2.84167}, {1.66, 2.87587}, {1.68, 2.91019}, {1.7, 2.94462},
 {1.72, 2.97915}, {1.74, 3.01378}, {1.76, 3.04851}, {1.78, 3.08333}, {1.8, 3.11825},
 {1.82, 3.15326}, {1.84, 3.18837}, {1.86, 3.22355}, {1.88, 3.25883}, {1.9, 3.29418},
 {1.92, 3.32962}, {1.94, 3.36514}, {1.96, 3.40073}, {1.98, 3.4364}, {2., 3.47214}}
```

```
ListLinePlot[solH, AspectRatio → Automatic,
 tracé de liste de ligne   rapport d'aspect   automatique
  AxesOrigin → {0, 0}, ImageSize → {300, 500}, AxesLabel → {"t", "y"}]
  origine des axes   taille d'image   titre d'axe
```